

Math 22 Unit Exam 3

Instructions. Read each problem carefully and follow all of its instructions.

Problems.

1. Find the exact value of $1 + e^{-1} + e^{-2} + e^{-3} + \dots + e^{-n} + \dots$
2. Use the integral test to determine whether $\sum_{n=1}^{\infty} \frac{n}{4n^2 + 5}$ converges.
3. Use the comparison test to determine whether $\sum_{n=0}^{\infty} \frac{1}{2^n + 3}$ converges.
4. Why are you able to use the alternating series test to determine whether $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 1}$ converges? Does it converge?
5. Use the ratio test to determine whether $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ converges.
6. Find the Taylor series for $\cos(\sqrt{x})$ about $x = 0$.
7. Find the interval of convergence for $\sum_{n=1}^{\infty} (-1)^n n^2 x^n$
8. Using Taylor series about $x = 0$, evaluate
$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x(e^x - 1)}.$$
9. If the radius of convergence of the power series $\sum_{n=0}^{\infty} C_n x^n$ is 10, what is the radius of convergence of the power series $\sum_{n=0}^{\infty} n C_n x^{n-1}$?