__Directions:__ Duration: 4 hours. One crib sheet is allowed. Each question is worth 10 points. **Credit will be awarded mainly based on the level of work and explanation (no explanation = no credit).**

1. Sketch the graphs the following functions (show your analysis):

$$(a)\ y(x) = \frac{x}{\sqrt{1-x^2}}\ ,\quad (b)\ y(x) = \ln\frac{x+1}{x-1}\ .$$

2. Find each limit if it exists. Otherwise, explain why it does not exist.

$$(a)\ \lim_{x\to 0}\sin\frac{1}{x}\ ,\quad (b)\ \lim_{x\to 0+} e^{\frac{1}{x}}\ln x\ ,\quad (c)\ \lim_{b\to a}\frac{b^5-a^5}{b-a}\ ,\quad (d)\ \lim_{x\to 0}\left(\frac{2}{\pi}\tan^{-1}x\right)^{\frac{1}{x}}\ .$$

3. Let $y(x)$ be defined implicitly by $(1+y)^3 = \ln(x+y)$. Evaluate $\frac{dy}{dx}$ when $y=0$.

4. Find the following indefinite integrals:

$$(a)\ \int\frac{e^x}{\sqrt{1+e^{2x}}}\,dx\ ,\quad (b)\ \int\frac{dx}{\sqrt{2-x}}\ ,\quad (c)\ \int e^x\cos 2x\,dx\ .$$

5. Determine whether the definite integrals exist. You do not need to find their values, but you must justify your answer.

$$(a)\ \int_1^\infty \sin\frac{1}{x}\,dx\ ,\quad (b)\ \int_0^1\frac{dx}{\cos(x-1)}\ ,\quad (c)\ \int_0^\infty \tanh^{-1}x\,dx\ .$$

6. Let $f(x)$ be a continuous function from the interval $[0\ 1]$ to itself, i.e., $f:[0\ 1]\to[0\ 1]$. Prove that there is a point $x_0\in[0\ 1]$ such that $f(x_0)=x_0$.

7. Find the second-order Taylor polynomials for:
   (a) $f(x) = \cos\sqrt{x-1}$ around $x_0 = 1$.
   (b) $f(x) = \dfrac{1}{1+x+y}$ around $(x_0, y_0) = (0,0)$.

8. Find all the critical points of the function $u(x,y) = x^2 - 3xy + y^2$ and classify these points as maxima, minima or saddle.

9. Use Gauss's Divergence Theorem to evaluate the integral $\int_S \vec{u}\cdot d\vec{S}$, where S is a sphere of radius R around the origin, $\vec{u}(x,y,z) = (x-y)\vec{i} + (y-z)\vec{j} + (z-x)\vec{k}$, and $(\vec{i},\vec{j},\vec{k})$ are the unit vectors along the (x,y,z) directions.

10. Suppose the Sun's surface is a sphere of radius R and consider the the half-sphere that is facing Earth, whose perimeter contour is denoted by C. Use Stokes' Theorem to evaluate the contour integral $\oint_C \vec{u}\cdot d\vec{r}$, where $\vec{u}(x,y,z) = xyz(\vec{i}+\vec{j}+\vec{k})$ and (x,y,z) is measured from the Sun's center.