TITLE: Nonlinear Schrödinger Equation with a White-Noise Potential

ABSTRACT: We propose a phase-space formulation for the nonlinear Schrödinger equation with a white-noise potential in order to shed light on two issues: the rate of spread and the singularity formation in the average sense. Our main tools are the energy law and the variance identity. The method is completely elementary.

For the problem of wave spread, we show that the ensemble-averaged dispersion in the critical or defocusing case follows the cubic-in-time law while in the supercritical and sub-critical focusing cases the cubic law becomes an upper and lower bounds respectively.

We have also found that in the critical and supercritical focusing cases the presence of a white-noise random potential results in different conditions for singularity-with-positive-probability from the homogeneous case but does not prevent singularity formation. We show that in the supercritical focusing case the ensemble-averaged self-interaction energy and the momentum variance can exceed any fixed level in a finite time with positive probability.