Numerical Solution of the Nonlinear Helmholtz Equation

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The nonlinear Helmholtz equation models the propagation of intense laser beams in Kerr media such as water, silica and air. It is a semilinear elliptic equation which requires non-selfadjoint radiation boundary-conditions, and remains unsolved in many configurations.

Its common approximation, the nonlinear Schrodinger equation (NLS), is known to lead to nonphysical singularities. We therefore consider the question, which has been open since the 1960s: *Do nonlinear Helmholtz solutions exists, under conditions for which the NLS solution becomes singular ?*

In this work we develop a numerical method which produces such solutions, thereby closing this question.

Numerically, we consider the case of grated material, that has material discontinuities in the direction of propagation. We develop a fourth-order discretization which is "semi-compact", i.e., compact only in one direction, that is optimal for this case.

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