- On the front of your blue book print (1) your name, (2) your student ID number, (3) your discussion section number, and (4) a grading table.
- Show all work in your blue book and BOX IN YOUR FINAL ANSWERS where appropriate.
- Please start each problem on a new page. There are a total of seven problems on both sides of this paper and a total of 100 points.
- NO books, notes, crib sheets, calculators or any other electronic devices are allowed.

Only the final answer will be graded for problem 1. No justification is needed.

1. (20 points, 4 points each) Determine whether the following statements are TRUE or FALSE. **Write out the whole word "TRUE" or "FALSE" for each problem.**

   (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}(2^{\sin(x)}) = \sin(x)2^{\sin(x)-1}\cos(x)$

   (b) The function $\cos(x)$ has all real numbers as domain and $[-1,1]$ as range, so its inverse function $\arccos(x)$ has $[-1,1]$ as domain and all real numbers as range.

   (c) $\dfrac{\mathrm{d}}{\mathrm{d}x}[\cosh^2(\sqrt{x}) - \sinh^2(\sqrt{x})] = 0.$

   (d) $\sin(\arctan(x)) = \dfrac{x}{\sqrt{1-x^2}}.$

   (e) If $f'(x)$ is defined for all x and f has a maximum at $x = 1$, then $f'(1) = 0$.

Show your reasoning clearly for problems 2–7. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit.

2. (24 points: 8 points each) Find the derivative of the following functions with respect to x.

   (a) $\dfrac{x-6}{x+7}$ (b) $(1+x^2)\arcsin(x)$ (c) $\tan(\ln(1-x))$

3. (9 points) Find the tangent line approximation to $\sqrt{1+x}$ at $x = 0$. Use this approximation to estimate $\sqrt{1.02}$.

4. (9 points) The part of the graph of
$$\sin(x^2 + y) = x$$
that is near $(0, \pi)$ defines y as a function of x implicitly. Is this function increasing or decreasing near $x = 0$? Explain how you know.

5. (7 points) Using the definition of the derivative, show that $\frac{\mathrm{d}}{\mathrm{d}x}\cos(x) = -\sin(x)$. You may need to use the following limits:
$$\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta} = 1 \qquad \text{and} \qquad \lim_{\theta\to 0}\frac{\cos(\theta)-1}{\theta} = 0.$$

6. (9 points: 4, 5)

   (a) Carefully state the Constant Function Theorem.

   (b) Suppose that $f(x)$ is differentiable for all x and that $f'(x) \leq 3$. If $f(0) = 4$, what can you say about the value of $f(2)$? Specify which theorem you are using.

7. (22 points total) Consider the function

$$f(x) = x^4 - 4x^3.$$

   Questions (a)–(f) will help you sketch the graph of $f(x)$.

   (a) (1 point) What is the domain of f?

   (b) (3 points) Is f even, odd, or neither? Why?

   (c) (5 points) Find $f'(x)$. On what interval(s) is f increasing? decreasing? Where are the local max/min points and what are the local max/min values?

   (d) (5 points) Find $f''(x)$. On what interval(s) is f concave up? concave down? Are there any inflection points? If there are, what are their coordinates?

   (e) (3 points) What are the x– and y–intercepts?

   (f) (2 points) What are the limits $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$?

   (g) (3 points) Sketch the graph of f. Make sure that it reflects your answers to all previous parts and mark the points from parts (c), (d) and (e) on your graph.