UC Merced: MATH 21 — Exam #3 — 07 December 2007

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your discussion section number and instructor's name (Sprague or Lei), (4) a grading table, and (5) your seat number. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. There are a total of FIVE problems on one side of this paper and a total of 50 points. Please start each of the FIVE problems on a new page. You have 50 minutes to complete the exam.

Some potentially useful information:

$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$; $\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$; $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$

For certain conditions, the following is true: $\left(f^{-1}\right)'(a) = \frac{1}{f'\left(f^{-1}(a)\right)}$

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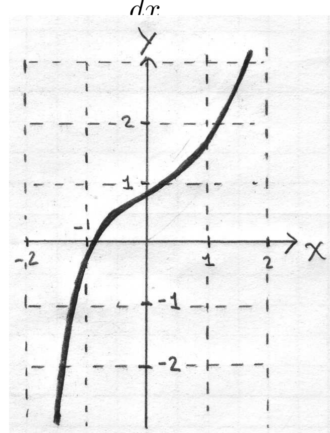
1. (6 points) Answer the following Always True (T) or False (F). Only your final answers will be graded on these problems.
   (a) The function $g(r) = 115^r + r^3$ is a one-to-one function.
   (b) $\lim_{x\to 0} \frac{\cos(x)}{x} = \lim_{x\to 0} \frac{-\sin(x)}{1} = 0$
   (c) Of the three hyperbolic functions we studied ($\sinh(x)$, $\cosh(x)$, $\tanh(x)$), only $\cosh(x)$ has a critical point.

2. (15 points) Complete the following differentiation problems:
   (a) Determine $\frac{dy}{dx}$ for $y(x) = \int_2^x \cosh(s)\log_4(s^2)\,ds$.
   (b) Using the knowledge that e^x is the inverse function of $\ln(x)$, show that $\frac{d}{dx}e^x = e^x$.
   (c) Determine $w'(x)$ if $w(x) = \ln\left[\arctan(x^2)\right]$.

3. (12 points) Complete the following integration problems:
   (a) $\int_1^4 \frac{1}{\sqrt{t}(1+\sqrt{t})}\,dt$
   (b) $\int \frac{1}{\sqrt{9-9w^2}}\,dw$

4. (12 points) Complete the inverse-function problems.
   (a) If $m(\theta) = \frac{4\theta - 1}{2\theta + 3}$, determine $m^{-1}(\theta)$.
   (b) Consider the graph to the upper right of the function $y(t)$. In your bluebook, copy (or trace) the graph and carefully sketch the graph of $y^{-1}(t)$.



Problem 4(b)

5. (5 points) A bacteria population has an initial population of 100. After 1 hour, the population is 400. What is the population after 10 hours? Clearly state your assumptions in determining your result.