

Name: \_\_\_\_\_ Signature: \_\_\_\_\_

Student ID: \_\_\_\_\_ Section number: \_\_\_\_\_

- On the front of your blue book print (1) your name, (2) your student ID number, (3) your discussion section number, and (4) a grading table.
- Show all work in your blue book and BOX IN YOUR FINAL ANSWERS where appropriate.
- Please start each problem on a new page. There are a total of — problems on both sides of this paper and a total of 100 points.
- NO books, notes, crib sheets, calculators or any other electronic devices are allowed.

**Show your reasoning clearly. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit.**

1. (23 points: 9, 7, 7)

$$f(x) = e^{x-1} + 2.$$

- (a) Sketch the graph of  $y = f(x)$ . Indicate clearly all intercepts and all asymptotes, if they exist.
- (b) What are the domain and range of  $f(x)$ ?
- (c) Is  $f$  invertible? If yes, find a formula for  $f^{-1}(x)$ . If not, explain why not.
2. (20 points) Consider the piecewise function  $f(x)$  defined below. Where is this function continuous? Where is it not continuous? Explain why.

$$f(x) = \begin{cases} \sqrt{1 + \cos(x)} & x \geq 0 \\ -2x, & x < 0 \end{cases}.$$

3. (20 points: 10 each) Find the following limits if they exist, either as numbers or  $\pm\infty$ . If a limit does not exist, explain why not.

(a)  $\lim_{x \rightarrow \infty} \frac{x^2 + 1}{x^3 - x + 1}$

(b)  $\lim_{x \rightarrow 1^+} \frac{1}{1 - x}$

4. (17 points: 10, 7)

$$g(x) = \sqrt{1 - x}.$$

- (a) Using the definition of a derivative, find  $g'(x)$ .
- (b) Find an equation of the tangent line to the graph  $y = g(x)$  at  $x = -3$ .
5. (20 points) Sketch a possible graph of  $y = f(x)$  if  $f(x)$  satisfies all of the following properties. Your answer should be a SINGLE graph.

- $f(x)$  is defined on the interval  $(0, \infty)$ .
- $\lim_{x \rightarrow 0^+} f(x) = \infty$ .
- $\lim_{x \rightarrow \infty} f(x) = 5$ .
- $f(x) = 0$  at  $x = 1$  and  $x = 3$ .
- $f'(x) < 0$  when  $0 < x < 2$ .
- $f'(x) > 0$  when  $2 < x$ .
- $f''(x) < 0$  when  $4 < x$ .
- $f''(x) > 0$  when  $0 < x < 4$ .