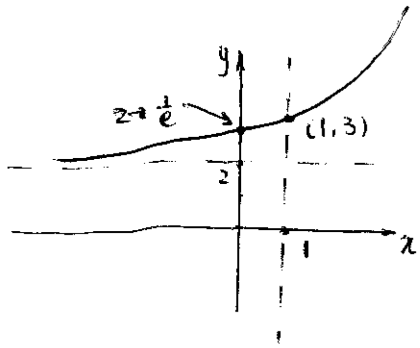
Math 21 : Calculus I — UC Merced, Spring 2007

Solutions to Midterm 1

1. (a)



$\boxed{\text{no } x\text{-intercept}}$

y-intercept : $x=0 \Rightarrow$

$y = e^{0-1}+2 = 2+\frac{1}{e}$

$\boxed{(0,\ 2+\frac{1}{e})}$

horizontal asymptote : $\boxed{y=2}$

$\boxed{\text{no vertical asymptote}}$

The graph of $y = e^{x-1}+2$ is the graph of $y=e^x$ shifted to the right by 1 and up by 2 units.

(b) domain : $\boxed{x \text{ can be all real numbers}}$

range : $\boxed{y>2}$

(c) f is invertible since its graph passes the horizontal line test.

Let $y = e^{x-1}+2$, then $y-2 = e^{x-1} \Rightarrow \ln(y-2) = x-1$

$\Rightarrow \ln(y-2)+1 = x \Rightarrow f^{-1}(y) = \ln(y-2)+1 \Rightarrow$

$$\boxed{f^{-1}(x) = \ln(x-2)+1}$$

2. All elementary functions are continuous in their domains.

$\sqrt{1+\cos(x)}$ and $-2x$ are both elementary functions defined for all real numbers, so $\boxed{f(x) \text{ is continuous at all } x \neq 0}$

At $x=0$:

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \sqrt{1+\cos(x)} = \sqrt{1+\cos(0)} = \sqrt{1+1} = \sqrt{2}$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0^-} -2x = -2(0) = 0$$

$\sqrt{2} \neq 0 \Rightarrow \lim_{x\to 0^+} f(x)$ does not exist. So $\boxed{f \text{ is not continuous at } x=0}$

3. (a) $$\lim_{x\to\infty} \frac{x^2+1}{x^3-x+1} = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{1}{x^3}}{1-\frac{1}{x^2}+\frac{1}{x^3}} = \frac{0+0}{1-0+0} = \boxed{0}$$

(b) $$\lim_{x\to 1^+} \frac{1}{1-x} = \frac{1}{0^-} = \boxed{-\infty}$$

4. (a) $$g'(x) = \lim_{h\to 0} \frac{\sqrt{1-(x+h)} - \sqrt{1-x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{1}-(\cancel{x}+h) - [\cancel{1}-\cancel{x}]}{\cancel{h}\left[\sqrt{1-(x+h)} + \sqrt{1-x}\right]}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{1-(x+h)} + \sqrt{1-x}}$$

$$= \frac{-1}{\sqrt{1-x} + \sqrt{1-x}}$$

$$= \boxed{\frac{-1}{2\sqrt{1-x}}}$$

(b) slope $= g'(-3) = \dfrac{-1}{2\sqrt{1-(-3)}} = -\dfrac{1}{2\sqrt{4}} = -\dfrac{1}{4}$

point: $x = -3$

$y = g(3) = \sqrt{1-(-3)} = \sqrt{4} = 2$

tangent line: $y - 2 = -\frac{1}{4}(x+3)$

$$y = -\frac{1}{4}x - \frac{3}{4} + 2$$

$$\boxed{y = -\frac{1}{4}x + \frac{5}{4}}$$

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