Math 21, UC Merced, Spring 2007

Solutions to Midterm 3

1. (a) $\lim_{x\to 0} x\ln(x) = 0\cdot\ln(0) = 0(-\infty)$ —— indeterminate

$$= \lim_{x\to 0} \frac{1}{\frac{1}{x}}\ln(x) = \lim_{x\to 0}\frac{\ln(x)}{\frac{1}{x}} = \frac{\ln(0)}{\frac{1}{0}} = \frac{-\infty}{\infty}$$

$$\overset{\text{L'Hopital}}{=} \lim_{x\to 0}\frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0}\frac{1}{\cancel{x}}\cdot\left(-\frac{x^{\cancel{2}}}{1}\right) = \lim_{x\to 0} -x = \boxed{0}$$

(b) $$\text{ave. value} = \frac{\int_0^2 (1+2x)\,dx}{2-0}$$

$$\int_0^2 (1+2x)\,dx = \int_0^2 1\,dx + \int_0^2 2x\,dx = x\Big|_0^2 + x^2\Big]_0^2$$

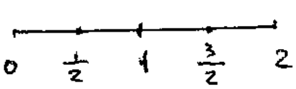
$$= (2-0) + (2^2-0^2)$$

$$= 2+4 = \boxed{6}$$

(c) f even $\Rightarrow \int_0^2 f(x)\,dx = \frac{1}{2}\int_{-2}^2 f(x)\,dx = \frac{1}{2}\cdot 10 = 5$

$$\int_2^5 f(x)\,dx = \int_{-2}^5 f(x)\,dx - \int_{-2}^2 f(x)\,dx = 4-10 = -6$$

$$\int_0^5 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^5 f(x)\,dx = 5-6 = \boxed{-1}$$

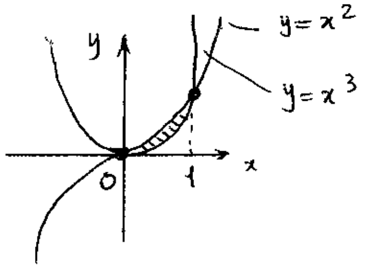
2. (a) 

left endpoints : $x = 0, \frac{1}{2}, 1, \frac{3}{2}$

width $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

$$\text{Riemann sum} = \boxed{\frac{1}{2}\left[\sin(0^2) + \sin\left(\left[\tfrac{1}{2}\right]^2\right) + \sin(1^2) + \sin\left(\left[\tfrac{3}{2}\right]^2\right)\right]}$$

(b)



intersection points: $x^2 = x^3 \Rightarrow x^3 - x^2 = 0$

$\Rightarrow x(x^2 - x) = 0 \Rightarrow x^2(x-1) = 0$

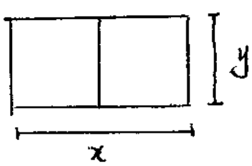
$\Rightarrow x = 0, \quad x = 1$

$$\text{area} = \boxed{\int_0^1 x^2\,dx - \int_0^1 x^3\,dx = \int_0^1 x^2 - x^3\,dx}$$

(c) $2006 - 1990 = 16$

price in 2006 = $\boxed{\int_0^{16} r(t)\,dt + 194.3 \text{ thousands of dollars}}$

3.



Find the global max. of $A = xy$.

$2x + 3y = 120 \Rightarrow 2x = 120 - 3y \Rightarrow x = 60 - \frac{3}{2}y$

$A = y \cdot (60 - \frac{3}{2}y) = 60y - \frac{3}{2}y^2$ and $0 < y < 120$.

$A' = 60 - 3y = 0 \Rightarrow y = 20$

$y = 20 \Rightarrow A = 20(60 - \frac{3}{2} \cdot 20) = 20(60 - 30) = 20(30) = 600$

$y = 0 \Rightarrow A = 0$

$y = 120 \Rightarrow A = 120(60 - \frac{3}{2} \cdot 120) < 0$

So, when $y = 20$, area A reaches max. of $\boxed{600 \text{ sq meters}}$