Exam # 2. Solution

1.

a) A car drives non-stop 400 mi in 5 hours on I-5 where the speed limit is 70 mph. After hearing this story a policeman can justify giving the driver a speeding ticket since the cars.

This is True. According to the Mean Value Thm if f(x) is continuous on an interval then somewhere in the interval the instantaneous rate equals to the average rate. In this problem f(x) is the displacement of the car and velocity is the rate.

b) If f'(x) = x cos(x) + sin(x) then f(x) = x sin(x) + 2x is a particular antiderivative.

This is False. f(x) = xsin(x) + 2 is a particular antiderivative. You can check it by taking a derivative of xsin(x)

c) If f'(c) = 0 and f''(c) < 0 then f has absolute maximum at c.

This is False. f has a local maximum at c.

d) Finding a number c where f'(c) = 0 guarantees that there is a local min or max at c.

This is False. f'(c) = 0 doesn't guarantee local max or min. Derivative of $y = x^3$ at x = 0 is a good example, (see Figure 9 on page 208).

e) If a smile is described by the function f(x) on [a; b] then f''(x) > 0 on (a; b).

This is True. A smile concaves up on interval (a,b) and this is precisely what f''(x)> 0 on (a,b) means. .

2. If a ball is thrown vertically upward, then its velocity after t seconds is given by v(t) = 200 - 20t (ft/sec). What is the height reached by the ball after 10 sec if the initial height is s(0) = 0?

To find height s(t) take the antiderivative of v(t). $s(t) = 200t – 10t^2 + c$. Since s(0) = 0 constant c = 0. After 10 sec.

s(10) = 2000 – 1000 = 1000 ft.

3. Function $y = \frac{100t}{t^2 + 25}$ describes fish population in a pond where t is the number of years ($t \geq 0$).

a) To find what will happen to fish population after a long time take the limit as time approaches infinity.

$$\lim_{t\to\infty} y = \frac{100t}{t^2+25} = \frac{\frac{100}{t}}{1+\frac{25}{t^2}} = \frac{0}{1} = 0$$ (Divide numerator and denominator by t^2)

The fish population will become extinct.

b) To find the maximum population, find critical t where y' = 0 and check for maximum.

Use the quotient rule to take derivative. Since the denominator $(t^2+25)^2$ is never zero an

$$\frac{dy}{dt} = \frac{100(t^2+25)-100t(2t)}{(t^2+25)^2} = \frac{2500-100t^2}{(t^2+25)^2} = 0$$

Since the denominator $(t^2+25)^2$ is never zero, we can set the numerator equal to zero.

$$2500 - 100t^2 = 0$$

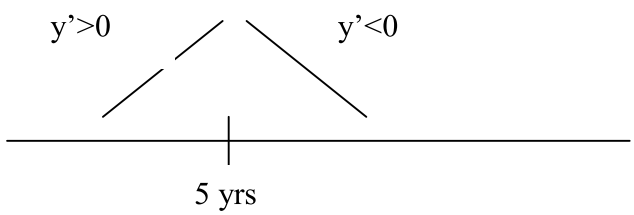
$$t = 5yrs$$

Since the domain t>0 we only need the positive root.

Now we must check for maximum. For that choose t to the right and to the left of t = 5.

When t = 4, $\frac{dy}{dt} = .5 > 0$

When t = 6, $\frac{dy}{dt} = -.295 < 0$



According to I/D test at t = 5 yrs we have an absolute maximum y' > 0 on interval [0,5) and y' < 0 on interval (5, ∞)

To find the maximum population calculate fish population at t =5,

y(5) = 10 fish

4. Calculate y' if $xy^4 = x + 3y$.

Take derivative with respect to x of the entire equation. You need to use the product rule on the xy^4 term.

$$x(4y^3\frac{dy}{dx})+\frac{dx}{dx}y^4=\frac{dx}{dx}+3\frac{dy}{dx}$$
$$4y^3x\frac{dy}{dx}+y^4=1+3\frac{dy}{dx}$$

Move dy/dx terms to the left and all other terms to the right.

$$4y^3x\frac{dy}{dx}-3\frac{dy}{dx}=1-y^4$$

Factor out dy/dx and then solve for dy/dx

$$\frac{dy}{dx}(4y^3x-3)=1-y^4$$
$$\frac{dy}{dx}=\frac{1-y^4}{4y^3x-3}$$

5. The area of a square is increasing at a rate of 10 cm^2/min. How fast is the perimeter increasing when the length of the side is 4 cm?

Area of the square is $A=x^2$ where x is the length of a side.
Perimeter on the square is P = x+x+x+x = 4x or x = P/4
Substitute x= P/4 into the expression for the area

$A= P^2/16$

Now take time derivative of the entire equation

$$\frac{dA}{dt}=\frac{2P}{16}\frac{dP}{dt}$$

Solve for dP/dt. When x=4 cm, P = 16 cm

$$\frac{dP}{dt}=\frac{8}{P}\frac{dA}{dt}=\frac{8}{16}10=5\frac{cm}{\sec}$$