UC Merced: MATH 21 — Final Exam — 09 May 2009

Instructions (Failure to follow these instructions may result in a loss of points.)
On the front of your bluebook print (1) your name, (2) your discussion section number, (3) your instructor's name (Crona, Lei, or Sprague), (4) your seat number, and (5) a grading table. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. There are a total of 9 problems on both sides of this paper and a total of 100 points. Please start each of the 9 problems on a new page. You have 3 hours to complete the exam.

Some potentially useful information:
$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$; $\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$; $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$

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1. (10 points: 2 each) Answer the following Always True (True) or False (False). Only your final answers will be graded on these problems.
   (a) $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)$ is undefined because $\lim_{x\to 0}\left(\frac{1}{x}\right)$ is undefined.
   (b) The function $g(x) = \int_0^x e^t |\sin(t)| dt$ is a one-to-one function.
   (c) $\int_{-1}^{1} \frac{2}{x^3}\, dx = -\frac{1}{x^2}\Big|_{-1}^{1} = 0.$
   (d) $y(x) = 0$ is a solution to $\frac{dy}{dx} = 5y$.
   (e) $\int \frac{3}{t}\, dt = 3\ln(t) + C$
2. (15 points: 5 each) Answer the following limit-type problems.
   (a) If it exists, determine the limit $\lim_{x\to 2}\frac{2+x}{e^x}$. If it does not exist, explain why.
   (b) If it exists, determine the limit $\lim_{x\to -2}\frac{2-|x|}{2+x}$. If it does not exist, explain why.
   (c) Show that $\lim_{x\to 0}\frac{\sin(x)}{x} = 1$
3. (20 Points: 5 each) Answer the following derivative-type problems
   (a) Using the definition of a derivative, calculate $\frac{dg}{dx}$ where $g(x) = \sqrt{x}$. **No credit** will be awarded for using short–cut formulas.
   (b) Use your knowledge of $\frac{d}{d\theta}\sin(\theta)$ and $\frac{d}{d\theta}\cos(\theta)$ to show that $\frac{d}{d\theta}\tan(\theta) = [\sec(\theta)]^2$.
   (c) Find an equation of the tangent line to the curve $x^2y^2 + y^3 = 2$ at the point $(1, 1)$.
   (d) Use your knowledge of the functions e^x and $\ln(x)$ to show that

$$\frac{d}{dx}(4^x) = 4^x \ \ln(4)$$

4. (15 points: 5 each) Answer the following integral-type problems.

   (a) Use a Riemann sum with 2 subintervals to approximate $\int_0^4 (x+1)^2 dx$ with the left-endpoint Rule.

   (b) Evaluate $\int_0^{\pi/2} \cos(x)\sin^2(x)\ dx$

   (c) Evaluate $\int_0^e w(x)\ dx$, where $w(x) = \begin{cases} 2-x & x<1 \\ \dfrac{1}{x} & x \geq 1 \end{cases}$

5. (6 points) Show that of all the rectangles with a given **perimeter**, the one with largest **area** is a square.

6. (5 points) The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

7. (9 points: 3 each) Let $v(t)$ denote the vertical speed of a Peregrine falcon as a function of the time t in minutes. Here $v(t) < 0$ if the falcon descends. Answer the following questions in terms of $v(t)$. Only your final $\boxed{\text{boxed}}$ answer will be graded.

   (a) Write an expression for the vertical displacement by the falcon from $t = 0$ to $t = 15$.

   (b) Write an expression for the vertical distance traveled by the falcon from $t = 0$ to $t = 15$.

   (c) The average acceleration of the falcon from $t = 0$ to $t = 15$.

8. (15 points: 3 each) Short answer; only your final $\boxed{\text{boxed}}$ answer will be graded.

   (a) Calculate $\dfrac{dm}{dt}$, given $m(t) = \cos\left(t^2 + e^t\right)$.

   (b) Find $w(z) = \int \left(z^2 + z^{-2}\right)\ dz$.

   (c) For what values of k does $y = \dfrac{x^3 + 3x^2 + 5}{4x + 1 + x^k}$ have a horizontal asymptote?

   (d) Write down a function $f(x)$ that satisfies the following: (i) continuous everywhere except $x = 2$, (ii) $\lim_{x\to 2^-} f(x) = +\infty$, (iii) $\lim_{x\to 2^+} f(x) = -\infty$. (Note: write down an actual function; no sketch required)

   (e) Let $f(x) = \int_0^x t^2 e^{t^3}\ dt$ for $x \geq 0$. Find $f'(2)$, that is $\left.\dfrac{df}{dx}\right|_{x=2}$. Answer with a **number**.

9. (10 points: 2 each) Short answer; only your final $\boxed{\text{boxed}}$ answer will be graded. The following questions refer to the figure below that shows $\dfrac{dg}{dx}$ for a function $g(x)$ that is continuous on $(0, 5)$.

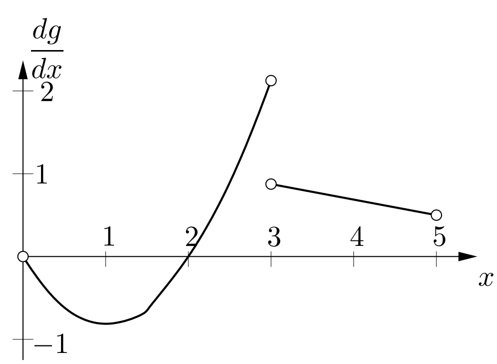
   (a) Over what interval(s) is $g(x)$ increasing?

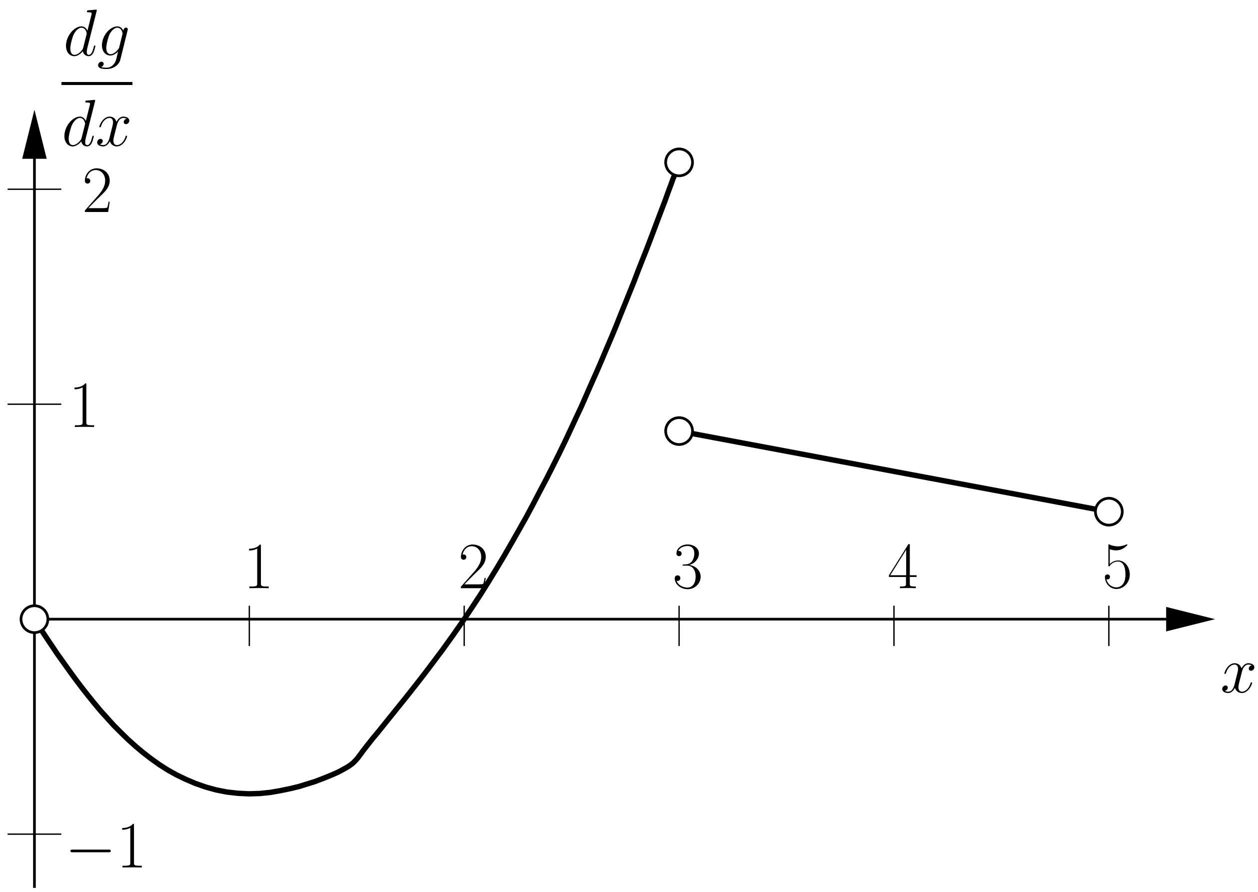
   (b) Over what interval(s) is $g(x)$ concave up?

   (c) Over what interval(s) is $g(x)$ concave down?

   (d) Does a local maximum of $g(x)$ exist? If so, where?

   (e) Does a local minimum of $g(x)$ exist? If so, where?




$\frac{dg}{dx}$
2
1
−1
1
2
3
4
5
x