Instructions. Attempt all questions. Answers must be justified in order to gain full credit. Calculators are not permitted.

1. Calculate the improper integrals if they converge.

   (i) (4 points) $\displaystyle\int_{-\infty}^{0} \frac{e^x}{1+e^x}\,\mathrm{d}x$ $\qquad$ (ii) (4 points) $\displaystyle\int_{0}^{2} \frac{1}{\sqrt{4-x^2}}\,\mathrm{d}x$

2. (5 points) Use the comparison test to decide whether the improper integral $\displaystyle\int_{1}^{\infty} \frac{2+\cos x}{x}\,\mathrm{d}x$ converges or diverges.

3. (9 points) Find the volume of the solid whose base is the region in the xy-plane bounded by $y = x^3$, $y = 1$, and the y-axis and whose cross-sections perpendicular to the x-axis are semicircles.

4. (9 points) Find the area inside the cardioid $r = 1 + \sin\theta$ and outside the circle $r = 1/2$.

5. (5 points) Find the arc length of the parabolic spiral $r = \theta^2$, $0 \le \theta \le \pi$.

6. A metal plate, with constant density 3 gm/cm^2, has a shape bounded by the curve $y = 2x^4$ and the x-axis, with $0 \le x \le 1$ and x, y in cm.

   (i) (4 points) Find the total mass of the plate.

   (ii) (10 points) Find $\bar{x}$ and $\bar{y}$.