Math 22: Final Exam

Spring Semester 2006

Instructions. Read each problem carefully and follow all of its instructions. For each of the problems below, write a clear and concise solution in your blue book. For any short answer questions, write clearly your answer and any additional explanation that is needed.

1. (10 points) The following formula appears often in mathematics and its applications

$$\int_0^\infty e^{-t^2}\,\mathrm{d}t = \frac{1}{2}\sqrt{\pi}$$

   Use this formula to evaluate

$$\int_m^\infty e^{-(t-m)^2/s^2}\,\mathrm{d}t$$

2. (10 points) Find $\displaystyle\int (1+\ln x)\sin^3(x\ln x)\,\mathrm{d}x$. (Hint: Use the substitution $w = x\ln x$).

3. (10 points) Find $\displaystyle\int \frac{u^2\,\mathrm{d}u}{\sqrt{9-u^2}}$.

4. (10 points) Find $\displaystyle\int \frac{\mathrm{d}x}{x^2-x-6}$.

5. Identify each of the following as TRUE or FALSE (*i.e.* not always true). No other justification is necessary.

   (a) (2 points) The integral $\displaystyle\int_0^{\pi/2} \ln(\sin x)\,\mathrm{d}x$ is improper.

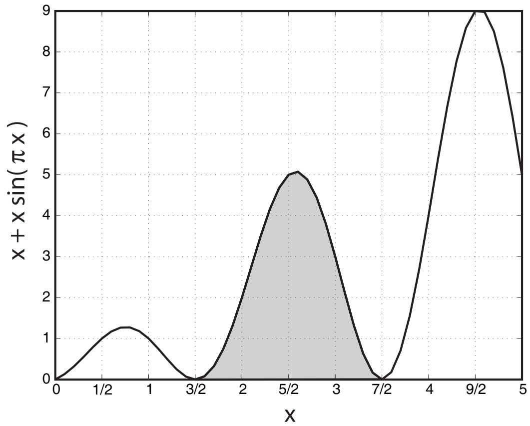
   (b) (2 points) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+3)^3}$ converges.

   (c) (2 points) The integral $\displaystyle\int_1^\infty \frac{x^2+x}{x^3+x^2+1}\,\mathrm{d}x$ diverges.

   (d) (2 points) If the sequence with general term s_n converges, $\displaystyle\lim_{n\to\infty} s_n = 0$.

   (e) (2 points) If $\displaystyle\lim_{n\to\infty} a_n = 0$, then the series $\sum a_n$ converges.

6. (10 points) The graph of the function $f(x) = x + x\sin(\pi x)$ is given below. Find the exact area of the shaded region.



7. (10 points) Solve the initial value problem

$$\frac{dx}{dt} = \frac{x}{2t} + xt, \quad x(1) = e.$$

8. Consider the function given by

$$f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{k!}{(2k)!}(x-2)^k$$

(a) (5 points) What is $f(2)$?

(b) (5 points) What is $f'(2)$?

(c) (5 points) What is $f''(2)$?

(d) (5 points) What is the Taylor expansion of $f(2x)$ about $x = 1$?

9. Give the function whose Taylor expansion is given below.

(a) (5 points) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{k!}x^k$

(b) (5 points) $\displaystyle\sum_{k=0}^{\infty} x^k$