Instructions

Attempt all questions. Answers must be justified in order to gain full credit. Calculators are not permitted. Turn this question sheet in with your blue book.

Time allowed: 50 minutes

Some useful trigonomteric identites:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

1. Evaluate the following integrals.

   (i) (5 points) $\displaystyle\int x \ln x \, \mathrm{d}x$ (ii) (5 points) $\displaystyle\int x^2 \sqrt{x-2} \, \mathrm{d}x$ (iii) (5 points) $\displaystyle\int \frac{\cos \sqrt{x}}{\sqrt{x}} \, \mathrm{d}x$

2. (10 points) By making a trigonometric substitution, evaluate the indefinite integral

$$\int \frac{x^2}{\sqrt{9 - 4x^2}} \, \mathrm{d}x.$$

3. (10 points) Find $\displaystyle\int x \sin^6 x^2 \, \mathrm{d}x$ using the reduction formula

$$\int \sin^n x \, \mathrm{d}x = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n}{n-1} \int \sin^{n-2} x \, \mathrm{d}x, \quad n \text{ positive.}$$

   Note: you will need to transfrom the integral first.

4. (15 points) Find the exact area of the region bounded by the curve $y = \dfrac{2x^2 + 3x + 3}{(x^2 + 1)(x + 2)}$, the x-axis, $x = 0$ and $x = 1$.