1. Evaluate the following integrals.

(i) $\displaystyle\int_0^1 x^2 e^x \, \mathrm{d}x$ (ii) $\displaystyle\int \frac{3x^3+13x^2+14x-5}{(x^2+2)(x+3)^2}\,\mathrm{d}x$ (iii) $\displaystyle\int \frac{e^x}{(1+e^x)^2}\,\mathrm{d}x$

(iv) $\displaystyle\int \frac{x^3+5x^2+5x+2}{x^3+2x^2+2x+1}\,\mathrm{d}x$ (v) $\displaystyle\int \ln(x^2)\,\mathrm{d}x$ (vi) $\displaystyle\int_0^{2/3} \frac{x^2}{\sqrt{16-9x^2}}\mathrm{d}x$

(vii) $\displaystyle\int_0^1 \frac{1}{(4-x^2)^{3/2}}\,\mathrm{d}x$ (viii) $\displaystyle\int (x+2)\sqrt{1-x}\,\mathrm{d}x$ (ix) $\displaystyle\int \frac{1}{x^2\sqrt{1+x^2}}\,\mathrm{d}x$

2. Derive the formula for the area of a circle with radius r using trigonometric substitution.

3. According to a book of mathematical tables,

$$\int_0^{\pi} \ln\left(5+4\cos(x)\right)\mathrm{d}x = 2\pi\ln(2).$$

Use this formula and the substitution $u = 4\tan^{-1}x$ to find the exact value of

$$\int_0^1 \frac{\ln\left(5+4\cos(4\tan^{-1}x)\right)}{1+x^2}\mathrm{d}x.$$

4. Calculate the following approximations to $\int_0^{\pi}\sin x\,\mathrm{d}x$.

(i) LEFT(4) (ii) RIGHT(4) (iii) MID(4) (iv) TRAP(4)