Instructions

Attempt all questions. Answers must be justified in order to gain full credit. Calculators are not permitted. Turn this question sheet in with your blue book.

Time allowed: 50 minutes

Some useful trigonomteric identites:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

1. Determine whether the integral is convergent or divergent. Evaluate those that are convergent.

   (i) (3 points) $\displaystyle\int_{2\pi}^{\infty} \sin x \, \mathrm{d}x$ (ii) (3 points) $\displaystyle\int_{2}^{5} \frac{1}{\sqrt{x-2}} \, \mathrm{d}x$

2. (5 points) Use the Comparison Test to determine whether the integral $\displaystyle\int_{1}^{\infty} \frac{2+e^{-x}}{x} \, \mathrm{d}x$ is convergent or divergent.

3. (8 points) Find the volume of the solid obtained by rotating the region enclosed by the curves $y = x$ and $y = x^2$ about the line $x = -1$.

4. (8 points) Find the volume of the solid whose base is the triangular region with vertices $(0,0), (1,0)$, and $(0,1)$ and whose cross-sections perpendicular to the y-axis are squares.

5. (i) (4 points) Sketch the polar curve $r = 1 + 2\cos\theta$.

   (ii) (8 points) Find the area of the inner loop of the limaçon in part (i).

6. A metal plate, with constant density 3 g/cm^2, has a shape bounded by the curve $y = 1 - x^2$ and the x-axis with x, y in cm.

   (i) (3 points) Find the total mass of the plate.

   (ii) (8 points) Find the center of mass $(\bar{x}, \bar{y})$ of the plate.