

1. (i) Calculate the approximations MID(4) and TRAP(4) to $\int_1^3 (1 - x^2) dx$.
(ii) Without evaluating the integral, determine if the approximation is an under-estimate or an over-estimate.
2. Calculate the integrals if they converge.
(i) $\int_{-\infty}^{\infty} \frac{1}{x^2 + 25} dx$ (ii) $\int_{\pi/4}^{\pi/2} \frac{\sin x}{\sqrt{\cos x}} dx$ (iii) $\int_3^6 \frac{1}{(4 - x)^2} dx$
3. Decide whether the improper integrals converge or diverge.
(i) $\int_2^{\infty} \frac{1}{\sqrt{1 + x^3}} dx$ (ii) $\int_0^{\pi} \frac{2 - \sin x}{x^2} dx$ (iii) $\int_0^{\infty} \frac{1}{e^x + 2^x} dx$
4. Find, by slicing, a formula for the volume of a cone of height h and base radius r .
5. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $x = 1$, and $y = 0$ about the axis $x = 2$.
6. Find the volume of the solid whose base is the region bounded by $y = 2e^x$, $x = 1$, and the lines $x = 0$ and $x = 1$ and whose cross-sections perpendicular to the x -axis are equilateral triangles.
7. Find the arc length of the parametric curve $x = \cos 3t$, $y = \sin 5t$ for $0 \leq t \leq 2\pi$.
8. Sketch the polar curve $r = \cos 2\theta$.
9. Find the area of the region that lies inside the cardioid $r = 1 - \sin \theta$ and outside the circle $r = 1/2$.
10. For what values of θ on the polar curve $r = \theta$, with $0 \leq \theta \leq 2\pi$, are the tangent lines horizontal? Vertical?
11. Find the arc length of the polar curve $r = 1/\theta$ for $\pi \leq \theta \leq 2\pi$.
12. The density of oil in a circular oil slick on the surface of the ocean at a distance r meters from the center of the slick is given by $\delta(r) = 50/(1 + r)$ kg/m².
(i) If the slick extends from $r = 0$ to $r = 10,000$ m, find a Riemann sum approximating the total mass of oil in the slick.
(ii) Find the exact value of the mass of oil in the slick.
(iii) Within what distance r is half the oil of the slick contained?
13. A metal plate, with constant density 5 gm/cm², has a shape bounded by the curve $y = \sqrt{x}$ and the x -axis, with $0 \leq x \leq 1$ and x, y in cm.
(i) Find the total mass of the plate.
(ii) Find \bar{x} and \bar{y} .