Final Exam, Math 22, Fall 2008, 12/13/08

Instructions: Write your name and section number. Draw grading table on the cover. Read each problem carefully and follow all of its instructions. For each of the problems below, write a clear and concise solution in your blue book. Solutions must be simplified as much as possible, no full credit for partially completed problems. When using tests for convergence identify the test and reason why series converges/diverges. **Blue books with torn or missing pages will not be accepted !**

1. (10 pts) Answer the following Always True(T) or False(F) . Only your final answers will be graded on these problems. Unless specified $\sum$ refers to infinite sum. (1 pt each)
   a. The Midpoint Rule will always provide and underestimate when approximating the area under a curve that is concave downward.
   b. Parametric equations are useful because they can describe a much greater variety of curves than standard Cartesian expressions.
   c. $\int_0^{\pi/2} \sec^2(x)dx$ is an example of improper integral.
   d. Function $r = \cos(6\theta)$ has a symmetry about the pole.
   e. If $a_n > 0$ and $a_n > a_{n+1}$ then $\sum a_n$ converges.
   f. Every bounded sequence converges.
   g. $\sum_{n=1}^{\infty}\left(-\frac{5}{4}\right)^{n-1} = \frac{4}{9}$
   h. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n!} = \frac{1}{e}$
   i. If $\sum a_n$ convergent then $\sum a_n^2$ also convergent
   j. Since $\sum n^{-1}$ divergent, Limit Comparison Test proves that $\sum n^{-1}\ln(n)$ also divergent.

2. (10 pts) Compute the integral $\int_0^{\ln(2)} e^x/(2+e^x)\,\mathrm{dx}$
3. (10 pts) Compute the integral $\int_4^{\infty} 1/(x^2-9)dx$
4. Sketch the region bounded by $y=x^3$, x=0 and y = 1. Find the volume of revolution when the region is revolved about the x-axis.
5. (10 pts) Find the length of the curve described by the function $y = \frac{x^2}{8} - \ln(x)$ from x =1 to x = 4.

6. (10 pts) Derive the given formula where n and a are constants.

$$\int x^n \cos(ax)dx = \frac{1}{a}x^n \sin(ax) - \frac{n}{a}\int x^{n-1}\sin(ax)dx$$

7. (10 pts) Find the area of one petal of the four petal rose r = 2sin(2θ)

8. (10 pts) Prove that $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}...$ is a divergent series.

9. (10 pts) For what values of x will the series $\sum_{n=1}^{\infty}\frac{x^n}{n^2}$, converge? (Hint: Don't forget the endpoints)

10. (10 pts) Find Taylor's expansion of $f(x)=\cos(x)$ at $a=\frac{\pi}{2}$. Calculate at least four nonzero terms.

Trig Identities:

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\sin^2(x) = \frac{1}{2}(1-\cos(2x))$$

$$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$$