Name:______________________________

Vector Calculus **Exam 2** **October 18th**

There are 7 problems and 144 points total. The point value of each question is indicated. *Read each question carefully!*

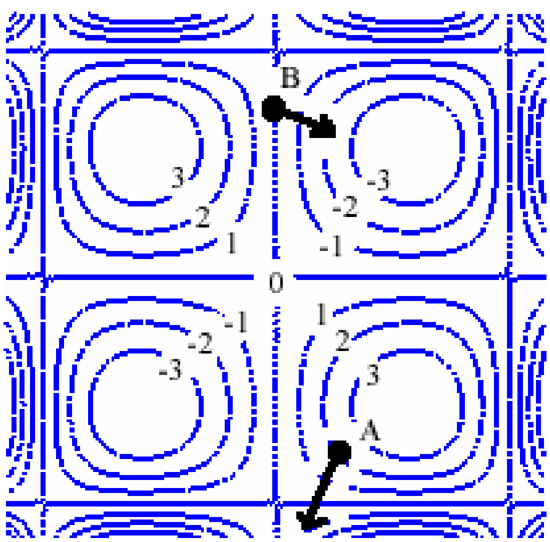
1. (24 points.) Let $f(x,y) = x^3 + 3xy^2$ and $g(x,y) = y^3 + 3x^2y$ Compute the following

a. $\frac{\partial}{\partial x}\left(f(x,y) + g(x,y)\right) =$

b. $\frac{\partial}{\partial y}\left(f(x,y) - g(x,y)\right) =$

c. $\nabla(2f(x,y)) =$

2. Use the figure below to answer the following questions



a) (3 points) What is the sign of the derivative at point A in the direction of the vector shown at A?

b) (3 points) What is the sign of the derivative at point B in the direction of the vector shown at B?

c) (6 points) Draw a vector in the direction of the gradient at point A

d) (6 points) Draw a vector in the direction of the gradient at point B.

3. (18 points.) . Find the derivative of $f(x,y)=\sqrt{9\cos(x)+16y}$ at the point $(x,y)=\left(0,1\right)$ in the direction $(3,5)$

4. (12 points.) A large metal plate is being chilled unevenly. The loss of heat causes each point (x,y) to have temperature $T(x,y)$ measured in ^{o}F . We know that $T(0,1)=5$, $T_x(0,1)=0$, and $T_y(0,1)=8/5$. Estimate the temperature at the point $(0.02,0.95)$

5. (12 points) Let $z = \sqrt{x^2 + y^2}$, $x = e^{\theta}$, and $y = e^{-\theta}$. Use the chain rule to compute $\dfrac{dz}{d\theta}$

6. Let $z = x^2 - y^2$, $x = \dfrac{u+v}{2}$, and $y = \dfrac{u-v}{2}$.

a) (16 points) Use the chain rule to compute $\dfrac{\partial z}{\partial u}$

b) (16 points) Use the chain rule to compute $\frac{\partial z}{\partial v}$

c) (4 points) Compute $\nabla z(u, v)$

7. Let $f(x, y) = e^{x^2 + y^2}$

a) (8 points) Find the critical points of f(x,y)

b) (16 points) Compute $f_{xy}(x, y)$ and $f_{yx}(x, y)$ and verify that $f_{xy}(x, y) = f_{yx}(x, y)$, Extra credit *Do not work on extra credit until you have finished the rest of the exam!*

A. (3 points) Draw a small circle around a saddle point in the contour diagram in problem **2**

B. (10 points) Show that wherever a level curve of $f(x,y)=\dfrac{y^2}{4x}$ intersects a level curve of $g(x,y)=2x^2+y^2$ the two curves are perpendicular to each other.