Instructions. Attempt all questions. Answers must be justified in order to gain full credit. Calculators are not permitted.

1. Let $f(x, y, z) = xe^y \sin z$.

   (a) Find a vector at the point $(0, 0, \pi/2)$ pointing in the direction in which f

   (i) (4 points) Increases fastest (ii) (2 points) Decreases fastest.

   (b) (4 points) Is there a direction at the point $(0, 0, \pi/2)$ in which the function does not change initially? If so, find a vector pointing in that direction.

2. (5 points) Find the directional derivative of $f(x, y, z) = xy + z^2$ at $(1, 1, 1)$ in the direction of $\vec{i} + 2\vec{j} + 3\vec{k}$.

3. (4 points) Use the chain rule to find $\partial z/\partial u$ and $\partial z/\partial v$ where

$$z = \cos(x^2 + y^2) \quad \text{with } x = u \cos v \text{ and } y = u \sin v$$

4. (4 points) Find the quadratic Taylor polynomial $Q(x, y)$ about $(0, 0)$ for the function $f(x, y) = \ln(1 + x^2 - y)$.

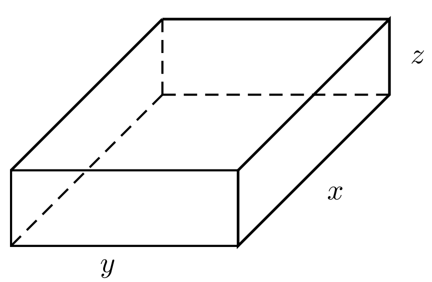
5. Let

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

   (a) (5 points) Is f differentialable at all points $(x, y) \neq (0, 0)$? Explain.

   (b) (5 points) Is f differentiable at $(0, 0)$? Explain.

6. A closed rectangular box has volume 32 cm^3.



   (a) (3 points) Find an expression for the surface area $S(x, y)$ of the box.

   (b) (2 points) What is the domain of S?

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(c) (5 points) Find the critical point of S and use the second derivative test to classify it.

(d) (2 points) Let $R = \{(x, y) \mid 1/3 \leq x \leq 288$ and $1/3 \leq y \leq 288\}$. Show that $S(x, y)$ is greater than the value of S at the critical point found in part (c) for points (x, y) outside the rectangle R.

(e) (5 points) Use part (d) and the extreme value theorem to explain why the critical point found in part (c) is a global minimum.