

**Duration: 50 minutes**

Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 50.

1. (10 points: 5 each) Given two vectors  $\vec{u} = \vec{i} - 2\vec{j} + 3\vec{k}$  and  $\vec{v} = \vec{j} + 2\vec{k}$ .
  - (a) Find an equation of the plane which is parallel to both  $\vec{u}$  and  $\vec{v}$  and goes through the point  $(2, 5, 3)$ .
  - (b) Decompose  $\vec{u}$  into two vectors  $\vec{a}$  and  $\vec{b}$  such that  $\vec{u} = \vec{a} + \vec{b}$ , with  $\vec{a}$  parallel to  $\vec{v}$  and  $\vec{b}$  perpendicular to  $\vec{v}$ .
2. (15 points: 5 each)
  - (a) Find parametric equations that represent the curve of intersection of the cylinder  $x^2 + y^2 = 9$  and the plane  $y + z = 1$ .
  - (b) Find the arc length of the helix  $\vec{r}(t) = \langle \sin 3t, 4t, \cos 3t \rangle, 0 \leq t \leq 2$ .
  - (c) Find parametric equations for the tangent line to the helix in part (b) at the point  $(0, 0, 1)$ .
3. (15 points total) Consider the function  $f(x, y) = \sqrt{x^2 + 4y^2 - 4}$ .
  - (a) (5 points) Draw a contour map of  $f$  showing at least 3 level curves. Remember to label your axes and level curves.
  - (b) (2 points) Draw 2 vertical traces of the graph  $z = f(x, y)$ , one with  $x = 0$  and the other with  $y = 0$ .
  - (c) (3 points) Sketch the graph  $z = f(x, y)$  showing your level curves and traces in parts (a) and (b).
  - (d) (5 points) Calculate  $f_x(1, 1)$  and  $f_y(1, 1)$ .
4. (10 points: 2 each) Answer the following questions in no more than two lines of text.
  - (a) A vector function  $\vec{r}(t)$  represents a space curve. If we know that  $\left| \frac{d\vec{r}}{dt} \right| = 1$  for all  $t$ , what is the geometric significance of the parameter  $t$  other than time?
  - (b) Is it true that if  $\vec{u} \times \vec{v} = \vec{0}$  then either  $\vec{u} = \vec{0}$  or  $\vec{v} = \vec{0}$ ? Explain why.
  - (c) How can you show that  $\lim_{(x,y) \rightarrow (a,b)} f(x, y)$  does not exist.
  - (d) Give an example of a function  $f(x, y)$  and a point  $(a, b)$  such that  $f_x(a, b)$  and  $f_y(a, b)$  both exist but  $f$  is not even continuous at  $(a, b)$ . You may describe your example using formulas, pictures or words.
  - (e) What is the length of the sum of two perpendicular unit vectors?