Duration: 3 hours

Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 100.

1. (20 pts: 2 each) Answer the following questions in no more than two lines of text or formulas (much less is usually needed if you are right on point).

   (a) Given vectors $\vec{a}$ and $\vec{b}$, describe in words, not formulas, the direction and magnitude of $\vec{a} \times \vec{b}$.

   (b) If you know that $\lim_{x\to 0} f(x, mx) = \lim_{x\to 0} f(x, kx^2)$, what can you conclude about $\lim_{(x,y)\to(0,0)} f(x, y)$?

   (c) At a given point, the gradient of $f(x, y)$ is $\nabla f = \vec{i} + \vec{j}$. In what direction would you have to move if you want to maintain a constant value of $f(x, y)$?

   (d) What is a formula for the average height of a surface $z = f(x, y)$ over a domain R in the xy-plane?

   (e) Given a vector function $\vec{r}(t)$ describing a space curve, how do you check whether or not t represents arc length?

   (f) If a level surface of $f(x, y, z)$ is given by $y^3 + x - z^2 = 1$, find a point where $f(x, y, z) = f(1, 0, 0)$ other than $(1, 0, 0)$.

   (g) When we change from rectangular coordinates (x, y, z) to spherical coordinates (ρ, ϕ, θ), what is the Jacobian?

   (h) Describe or sketch the parametrized surface $x = 2\cos\theta$, $y = t$, $z = 2\sin\theta$.

   (i) If S is a sphere and $\vec{F}$ is a vector field whose components have continuous partial derivatives on $\mathbb{R}^3$, why is $\iint_S \operatorname{curl} \vec{F} \cdot d\vec{S} = 0$?

   (j) Sketch or describe in words a vector field with a positive curl everywhere (a formula is not sufficient).

2. (7 pts) Given the vector $\vec{n} = 2\vec{i} - \vec{j} + \vec{k}$,

   (a) Find an equation of the plane perpendicular to $\vec{n}$ and going through the point $(0, 2, 4)$.

   (b) Decompose the vector $\vec{v} = -\vec{j} + \vec{k}$ into two parts $\vec{a}$ and $\vec{b}$ such that $\vec{v} = \vec{a} + \vec{b}$, with $\vec{a}$ parallel to $\vec{n}$ and $\vec{b}$ perpendicular to $\vec{n}$.

3. (9 pts) Consider the function $f(x, y) = 9x^2 + y^2 - 1$.

   (a) Draw a contour map of f showing at least 3 level curves. Remember to label your axes and level curves.

   (b) Draw 2 vertical traces of the graph $z = f(x, y)$, one with $x = 0$ and the other with $y = 0$.

   (c) Sketch the graph $z = f(x, y)$ showing your level curves and traces in parts (a) and (b).

4. (9 pts) Consider the function $f(x, y) = e^{2x} \sin y$.
   (a) In which direction does $f(x, y)$ **decrease** the fastest at $(0, \pi/4)$?
   (b) Find the directional derivative of f in the direction of $< 1, -1 >$ at $(0, \pi/4)$.
   (c) If x and y depend on another variable u: $x(u) = u^3$ and $y(u) = \pi/4 + 3u$, compute $\frac{df}{du}$ at $u = 0$.

5. (9 pts) Consider the function $f(x, y) = x^2 - y + y^2$.
   (a) Find and classify all critical points of $f(x, y)$.
   (b) Find the absolute maximum and absolute minimum values of $f(x, y)$ over $D = \{\, (x, y) \,|\, x^2 + y^2 \leq 1 \,\}$.

6. (10 pts) Consider the parametric curve

$$C: \quad x = \sin t, \qquad y = t, \qquad z = \cos t.$$

   (a) Find parametric equations of the tangent line to the curve C at the point $(0, 0, 1)$.
   (b) Evaluate the line integral of $\vec{F}(x, y, z) = z\vec{i} + y\vec{j} - x\vec{k}$ along C from $(0, 0, 1)$ to $(0, \pi, -1)$.

7. (7 pts) Consider $\vec{F}(x, y) = (1 - ye^{-x})\vec{i} + (e^{-x} + 5)\vec{j}$.
   (a) Find a function f such that $\vec{F} = \nabla f$.
   (b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is any path from $(0, 1)$ to $(1, 2)$.

8. (11 pts) D is a triangular region in the xy–plane with vertices $(0, 0)$, $(1, 0)$, and $(1, 2)$.
   (a) Set up **two** iterated integrals to evaluate $\iint_D f(x, y)\, dA$, one integrating x first and the other integrating y first.
   (b) Use Green's Theorem to evaluate $\oint_C (e^y + \cos x)\, dx + (3x^2 y + xe^y)\, dy$ where C is the boundary of D oriented counterclockwise.
   (c) Is $\vec{F}(x, y, z) = (e^y + \cos x)\vec{i} + (3x^2 y + xe^y)\vec{j}$ a conservative field? Why or why not?

9. (9 pts) Let S be part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the square $0 \leq x \leq 1$, $0 \leq y \leq 1$, and has upward orientation. Find the flux of $\vec{F}(x, y, z) = y\vec{i} - x\vec{j} + z\vec{k}$ across S.

10. (9 pts) Let S be the surface of the solid E bounded by the cone $z = \sqrt{(x^2 + y^2)}$ and the plane $z = 3$ oriented outward. Use the divergence theorem to calculate the flux of $\vec{F}(x, y, z) = (\sin y - 2xz)\vec{i} + (e^x - e^z + y)\vec{j} + (z^2 + 1)\vec{k}$ across S.

HAVE A GOOD SUMMER!!