1 (a) TRUE (b) TRUE (c) FALSE (d) FALSE (e) TRUE

2

(a) THREE LINEARLY INDEPENDENT VECTORS IN $\mathbb{R}^3$ ARE REQUIRED TO FORM A BASIS IN $\mathbb{R}^3$. $\underline{v}_1$, $\underline{v}_2$, AND $\underline{v}_3$ ARE IN $\mathbb{R}_3$, NEED TO TEST IF THE ARE LINEARLY INDEPENDENT.

LET $A = [\underline{v}_1 \ \underline{v}_2 \ \underline{v}_3] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$\det A = (1)(4-1) - (-1)(0-1) = 3 - 1 = 2 \neq 0 \Rightarrow$ LINEARLY INDEPENDENT

$\boxed{\underline{v}_1, \underline{v}_2, \underline{v}_3 \text{ FORM A BASIS FOR } \mathbb{R}^3}$

(b) LET $f_1, f_2 \in W$, AND $c_1, c_2 \in \mathbb{R}$

CHECK WHETHER OR NOT $(c_1 f_1 + c_2 f_2)$ IS IN $W$

$$\frac{d^4}{dx^4}(c_1 f_1 + c_2 f_2) + b(c_1 f_1 + c_2 f_2) = c_1 \underbrace{\left[\frac{d^4 f_1}{dx^4} + b f_1\right]}_{0} + c_2 \underbrace{\left[\frac{d^4 f_2}{dx^4} + b f_2\right]}_{0} = 0$$

CLEARLY, $f = 0$ IS IN $W$

$\boxed{W \text{ IS A SUBSPACE OF } C^4}$

(c) IF FUNCTIONS IN THE SET $\{t, t-1, t^2+1\}$ ARE LINEARLY INDEPENDENT, THEY WILL SPAN $\mathbb{P}_2$, WHOSE DIMENSION IS 3. USE WRONSKIAN

$$W[t, t-1, t^2+1] = \begin{vmatrix} t & t-1 & t^2+1 \\ 1 & 1 & 2t \\ 0 & 0 & 2 \end{vmatrix} = 2 \neq 0 \Rightarrow \text{LINEARLY INDEPENDENT}$$

$\boxed{\text{DIMENSION OF SUBSPACE IS THREE}}$

3 (a) STANDARD FORM: $\frac{d^2u}{dx^2} + 3\frac{du}{dx} + u = 0$ LET $u(x) = e^{rx}$

CHARACTERISTIC EQ'N $\boxed{r^2 + 3r + 1 = 0}$

ROOTS $r_{1,2} = \frac{-3 \pm \sqrt{9-4}}{2}$ $\boxed{r_1 = \frac{-3+\sqrt{5}}{2}, \ r_2 = \frac{-3-\sqrt{5}}{2}}$

(b) GENERAL SOLN: $\boxed{u(x) = C_1 \, \mathrm{EXP}\left[\frac{-3+\sqrt{5}}{2}x\right] + C_2 \, \mathrm{EXP}\left[\frac{-3-\sqrt{5}}{2}t\right]}$

WHERE $C_1, C_2$ ARE CONSTANTS

(c) FOR UNIQUE SOL'N, FIND $C_1, C_2$ TO SATISFY IC's

$$\left.\begin{aligned} u(0) &= C_1 + C_2 = 1 \\ u'(0) &= \left(\frac{-3+\sqrt{5}}{2}\right)C_1 + \left(\frac{-3-\sqrt{5}}{2}\right)C_2 = 0 \end{aligned}\right\} \quad \boxed{\begin{aligned} C_1 &= \frac{5+3\sqrt{5}}{10} \\ C_2 &= \frac{-3+\sqrt{5}}{2\sqrt{5}} \end{aligned}}$$

4 (a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ $\boxed{AA^T = \begin{bmatrix} 14 & 32 \\ 32 & 77 \end{bmatrix}}$

(b)

$$A = \begin{bmatrix} 2 & 4 & 0 \\ 4 & 9 & -3 \\ -2 & 0 & -7 \end{bmatrix} \quad \underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \underline{b} = \begin{bmatrix} 2 \\ 8 \\ 10 \end{bmatrix}$$

(c)

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 1 \\ -2 & 2 & 1 & 0 \\ 2 & -4 & 1 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ -2 & 2 & 1 & 0 \\ 2 & -4 & 1 & 1 \end{array}\right] \xrightarrow[-2R_1+R_3 \to R_3]{2R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ 0 & 2 & 2 & 1 \\ 0 & -4 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & 1 & 1/2 \\ 0 & -4 & 0 & 0 \end{array}\right] \xrightarrow{4R_2+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & 4 & 2 \end{array}\right]$$

$$\xrightarrow{\frac{1}{4}R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{array}\right] \xrightarrow[R_1 - \frac{1}{2}R_3 \to R_1]{R_2 - R_3 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \end{array}\right]$$

$\boxed{\underline{x} = (1/4, 0, 1/2)^T}$

WE FOUND THAT A IS ROW EQUIVALENT TO $I_3$, THUS $\boxed{\det A \neq 0}$

(d) $\det B = K + 2 \Rightarrow$ FOR ALL $K \neq -2$, $B^{-1}$ EXISTS.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ -1 & K & 0 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 2+K & 1 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{1}{2+K} & \frac{1}{2+K} \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & \frac{K}{2+K} & \frac{-2}{2+K} \\ 0 & 1 & \frac{1}{2+K} & \frac{1}{2+K} \end{array}\right]$$

$\boxed{B^{-1} = \frac{1}{2+K}\begin{bmatrix} K & -2 \\ 1 & 1 \end{bmatrix}, K \neq -2}$

5 LET $R(t)$ BE THE NUMBER OF RABBITS IN THE FOREST AT TIME $t$. THEN

$\frac{dR}{dt} = K\sqrt{R}$ FOR SOME CONSTANT $K$