**MATH 24 – EXAM 2** **2:00-3:30pm March 22nd, 2006**

ON THE FRONT OF YOUR BLUEBOOK WRITE (1) YOUR NAME, (2) A FIVE-PROBLEM GRADING GRID. **Show ALL of your work** in your bluebook, and **box in your final answers.** A correct answer, but without the relevant work, will receive no credit. This exam is closed-book and no calculators are allowed. You are allowed a one-page crib sheet. Start each problem on **the top of a new page.** Each problem is worth 20 points, for a total of 100 points. You can solve the problems in any order you like.

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1. Consider the linear system

$$\begin{aligned} 2x - 10z &= 0 \\ y + 2z &= 1 \\ -4x + 3y + 26z &= 3 \end{aligned}$$

   (a) Write this system in the augmented form $[A|\mathbf{b}]$.

   (b) Using the determinant, determine whether the system has a unique solution.

   (c) Solve this system using Gauss Elimination (RREF).

   (d) What is the span of the solutions (point, line, plane, ...) and what is its dimension?

2. Let $A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

   (a) Calculate $AA^T$.

   (b) Find $A^{-1}$.

   (c) Solve $A\mathbf{x} = \mathbf{b}$ when $\theta = \frac{\pi}{2}$.

   (d) Which of the following products are defined:
   (i) $A\mathbf{b}$, (ii) $\mathbf{b}A$, (iii) $A^T\mathbf{b}^T$, (iv) $\mathbf{b}^TA$, (v) $\mathbf{b}^T\mathbf{b}$?
   Note: you do not need to calculate these products.

3. Consider the following system of equations in augmented form

$$[A|\mathbf{b}] = \left[\begin{array}{cc|c} k & -2 & 1 \\ 2 & -k & -1 \end{array}\right]$$

   (a) For which values of $k$ does this system have a unique solution?

   (b) Determine the number of solutions of this system for all values of $k$.

   (c) Define the properties of a basis of a vector space. Explain whether the column vectors of $A$ form a basis of $\mathbb{R}^2$ when (i) $k = 0$ and (ii) $k = 2$.

4. Determine whether the following statements are TRUE or FALSE. Note: you must write the entire word TRUE or FALSE. You do not need to show your work for this problem.

   (a) The RREF of $A = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 0 & 10 & 0 & 1 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & -3 \end{bmatrix}$ is the identity matrix.

   (b) The vectors $\{[-3,1]\,,\,[2,-3]\,,\,[2,1]\}$ span $\mathbb{R}^2$.

   (c) The vectors $\{[-3,1]\,,\,[2,-3]\,,\,[2,1]\}$ form a basis of $\mathbb{R}^2$.

   (d) The functions $\{t^2, t^2+1, t^2+3t+1, t+4\}$ are linearly independent.

   (e) The set of solutions of the equation $x^2y'' + xy' - y = 0$ forms a vector space.

5. Consider the differential equation $y'' + y' - 6y = 0$.

   (a) Find the characteristic equation and its roots.

   (b) What is the general solution of this differential equation?

   (c) Solve the equation when $y(0) = 0$ and $y'(0) = 5$.

   (d) What is the long time behavior of the solution?

THE END