

## EXAM 1, MATH 24, FALL 2008

Instructions: Do not begin the exam until you are instructed to do so. You may write on the exam sheet, but ONLY what is written in your bluebook will be graded.

For each problem, you must show all work in order to receive credit. Partial credit will be given when appropriate, even if the final answer is not correct, but an answer with no work shown will receive zero credit regardless of correctness. You may not use any text, notes, or calculators on this exam, and collaboration is not allowed. Values can be expressed in symbolic form when appropriate, e.g.  $2\pi$ ,  $e^2$ ,  $\sin 1$ , etc. Decimal representations are not required. However, trigonometric functions of basic angles, e.g.  $\tan \frac{\pi}{4}$ ,  $\sin \pi$ , etc. should be evaluated.

1. (10 points each) Classify the following differential equations by determining (i) if they are linear/nonlinear, (ii) their order, (iii) if they are homogeneous/inhomogeneous (iv) if they have constant/variable coefficients. Parts (ii), (iii), and (iv) should only be answered if the equations are linear. Find the general solution to the DE **if** it is linear and first-order.

a.  $y' + 2ty = 2t$

b.  $y'' + t^2 = 0$

c.  $yy' = 8te^t$

d.  $y' - \frac{y}{t} = 0$

2. (5 points) For the direction field of a first-order differential equation  $y' = f(t, y)$ , explain why ALL horizontal lines are isoclines if  $f(t, y)$  is independent of  $t$ , that is, if  $f(t, y) = f(y)$ .

3. (30 points) The half-life of the 235-isotope of Uranium ( $^{235}\text{U}$ ) is approximately  $7 \times 10^8$  years. *Your answers must be numeric (i.e. no variables), but you do NOT have to carry out the arithmetic for either part of this problem.*

(a) What is the decay constant ( $k$ ) for  $^{235}\text{U}$  ?

(b) How long will it take for a sample of  $^{235}\text{U}$  to decay to one-tenth of its original amount?

4. (25 points) Solve the initial value problem  $\begin{cases} y' + \frac{2y}{t} = t + 1 \\ y(2) = 2 \end{cases}$ .