EXAM 3, MATH 24, FALL 2008

Instructions: Do not begin the exam until you are instructed to do so. You may write on the exam sheet, but ONLY what is written in your bluebook will be graded.

For each problem, you must show all work in order to receive credit. Partial credit will be given when appropriate, even if the final answer is not correct, but an answer with no work shown will receive zero credit regardless of correctness. You may not use any text, notes, or calculators on this exam, and collaboration is not allowed.

1. For the linear system

$$\begin{aligned} 4x - 2y &= 0 \\ 6x + 3y &= 0 \\ 2x - y &= 0 \end{aligned}$$

a. **(5 points).** Write the system in matrix form.

b. **(10 points).** Find a basis for the solution space.

2. A 1-kg mass is attached to a spring with restoring constant 4 N/m and placed on a horizontal surface. The spring is compressed 1 m, then (at $t = 0$) given a "push" (which compresses it further) such that it is given an initial velocity of 2 m/s. Neglect all forms of resistance for parts (a)−(c).

a. **(5 points).** Write down the initial value problem that models the position of the mass.

b. **(5 points).** What is the circular frequency of the mass-spring system, including proper units?

c. **(10 points).** Find the position of the mass as a function of time for $t > 0$, i.e. solve the IVP from part (a).

d. **(10 points).** If resistance is introduced to the system such that the damping constant is 5 kg/s, what would be the general solution for the position as a function of time? You do not need to consider the initial conditions.

e. **(5 points).** What is the long-term behavior of the system (i.e. as $t \to \infty$) in part (d)?

3. **(10 points).** Find the general solution of the system $\mathbf{x}' = A\mathbf{x}$, where $A = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$.

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4. (20 points). Find the general solution to the following DE.

$$y'' + 4y' + 4y = 9e^t$$

5. Explain the validity of the following statements with a short answer, example/counterexample, etc.

a. **(5 points).** Every simple (i.e. distinct, non-repeated) eigenvalue of a non-singular matrix corresponds to a unique eigenvector.

b. **(5 points).** If a matrix transforms a vector such that the resulting vector remains in the same vector space as the original, then the original vector is an eigenvector of the transformation matrix.

c. **(5 points).** An $n \times n$ linear system $A\mathbf{x} = \mathbf{0}$ is always consistent, and therefore a solution always exists. If the solution is unique, it is only the trivial solution $\mathbf{x} = \mathbf{0}$.

d. **(5 points).** If a 2nd-order linear system of (first-order) DE's has a repeated eigenvalue, then the eigenvectors cannot possibly span the eigenspace, and you must use a generalized eigenvector (i.e. $\mathbf{u}$ such that $(A - \lambda I)\mathbf{u} = \mathbf{v}$, where λ is an eigenvalue, and $\mathbf{v}$ is its eigenvector) to obtain 2 linearly independent eigenvectors.