

MATH 24 – Exam 2

Spring Semester 2008

Instructions. Please provide clear and concise solutions to the problems in this exam in your bluebook. Where applicable, explain your reasoning using complete sentences with proper grammar. Poor presentation may result in loss of credit.

1. (10 pts) Solve the linear system of equations

$$x_1 + x_2 + x_3 = -2$$

$$3x_1 + 3x_2 - x_3 = 6$$

$$x_1 - x_2 + x_3 = -1$$

2. For each of the two items below, please write a clear and concise written response.

(a) (5 pts) Explain why a 5×7 matrix never has linearly independent columns.

(b) (5 pts) Explain why there is exactly one solution of $A\vec{x} = \vec{b}$ for every \vec{b} when the columns of A are linearly independent.

3. (10 pts) Find the inverse of

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

and verify directly that $A^{-1}A = I_2$ and $AA^{-1} = I_2$ where I_2 is the 2×2 identity matrix.

4. (10 pts) Compute $\det(A)$ where

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{bmatrix}$$

5. (10 pts) Is the following set of vectors in \mathbb{R}^4 linearly independent? Provide a complete explanation of your answer.

$$S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ 1 \\ 4 \end{bmatrix} \right\}$$