Shape Optimization of Swimming Sheets

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The swimming behavior of a sheet which moves via wave propagation was first studied by G. I. Taylor in 1951. In addition to being of theoretical interest, this problem serves as a useful model of the locomotion of various micro-organisms and a few larger animals such as snails. We show how the shape of the wave affects the swimming speed and power required to swim, and present numerical techniques to find the fastest and most efficient swimming strategies when arclength and fluid volume are constrained. For each optimization problem, we obtain a one parameter family of solutions that become singular in a self-similar fashion as the parameter approaches a critical value. We explore the validity of the lubrication approximation used to model the dynamics by monitoring higher order corrections to the zeroth order theory. These corrections are themselves validated by comparison with finite element solutions of the full Stokes equations.