Duration: 240 minutes
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 100.

1. (20 pts) Evaluate the following limits using any method of your choice (but providing a justification).
   (a) $\lim_{x\to 0} \frac{\sin^3 x}{(e^{2x}-1)^3}$
   (b) $\lim_{x\to 0} \frac{e^{xy}}{y^x}$ for $y \in \mathbb{R}$
   (c) $\lim_{n\to\infty} \sum_{j=0}^{n} \frac{\sin(j/n)}{n}$
   (d) $\lim_{(x,y)\to(0,0)} \frac{x^2+y^4}{x^4+y^4}$
2. (15 pts) For each of the given equations, calculate $\frac{dy}{dx}$
   (a) $y(x) = \int_0^\infty e^{-xs} f(s)ds$ with $f(s) < 1/s$ as $s \to \infty$.
   (b) $y(x) = \int_{x^2}^{\sin x} e^{-xy^2} dy$
   (c) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
3. (20 pts) Compute the following integrals
   (a) $\int x \ \cos(\arcsin(x))dx$ assuming that x is between 0 and 1.
   (b) $\int \frac{x^2+1}{x^2-1}dx$
   (c) $\int_0^\infty e^{-x}x^n dx$ (try $n = 0$ first).
   (d) $\int_0^1 x^p dx$ as a function of the real number p
4. (10 pts) Compute $\sum_{n=0}^{\infty} nr^n$ as a function of r.
5. (10 pts) Find the first three non-zero terms of the Taylor Series of $f(x) = \cosh(x)e^{x^2}$ and find the radius of convergence of the Series.
6. (15 pts) Compute the volume of the shell-like body obtained by rotating the curves $y = 1-x^2$ and $y = 1 - x^4$ around the x-axis.
7. (15 pts) An airplane is collecting dust particles in a 4m^2 filter that is held perpendicular to the trajectory of the plane. The plane starts at the point $(1, 0, 0)$ and flies on a helical going in circles centered at the origin of radius 10 km, and rising by 1 km at very loop. If the density of pollutants if $f(x, y, z) = e^{-z}$g/m^3, how much dust will the plane have collected when it reaches a height of 5 km?
8. (10 pts) Determine for which values of x is the following Series convergent $\sum_{n=0}^{\infty} \frac{x^n}{(1+x^{2n})n}$
9. (10 pts) Consider a function of 2 variables $f(x, y)$ and suppose that $(0, 0)$ is a critical point of $f(x, y)$ and that $f(0, 0) = 0$. Using a Taylor Series expansion and square completion, rederive the criteria used to classify the critical point as a local minimum, local maximum or saddle point. You may assume that $f_{xx}(0, 0) \neq 0$ and $f_{yy}(0, 0) \neq 0$.
10. (10 pts) Compute $\int_0^1 \int_{y^{1/3}}^1 3\cos(x^4)\ dx\ dy$.

11. (15 pts) The mean geometric radius, R, of an ellipsoid given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is given by the radius of a sphere of volume equal to that of the ellipsoid. Give a formula for R as a function of a, b, and c. Then, find a linearization of the mean geometric radius about the point $R = 2$ corresponding to $a = 1/2$, $b = 1$, $c = 4$.

12. (15 pts) Compute $\int_C \vec{F} \cdot \frac{d\vec{r}}{dt} dt$ if $\vec{F} =< y^2 \sin x + x^2, -2y \cos x + \cosh y + xy >$ and C is the upper half-circle of radius 3 centered at the origin traced counterclockwise. (Hint: Use Green's theorem).

13. (15 pts) Consider the surface parametrized by
$x(u, v) = u \cos v$
$y(u, v) = u \sin v$
$z(u, v) = u$
for $0 \leq u \leq 1$ and $0 \leq v \leq 2\pi$.

    (a) Sketch or describe this surface.

    (b) Find its surface area.

    (c) Give a vector perpendicular to its surface for any choice of (u, v).

14. (10 pts) Let $\vec{F} =< -x + y, -x - y, -z >$ be the amount of solar wind per unit area, with $(0, 0, 0)$ being at the center of the Earth. How much solar wind enters the atmosphere if we assume that the atmosphere has radius 7 (thousand km).