Instructions: Credit will be awarded mainly based on the *quality of your work* and the *clarity of your explanations*.

1. Solve each of the following initial value problems.

   (a) $y' = xy^3(1+x^2)^{-1/2}, \quad y(0) = 1.$

   (b) $y' = e^{2x} + y - 1, \quad y(0) = 0.$

   (c) $y'' + 4y = 3\sin 2x, \quad y(0) = 2, \quad y'(0) = 1.$

2. Consider the ODE
$$\dot{y} = e^{-y}\sin y.$$

   (a) What are the fixed/equilibrium points?

   (b) Determine the stability of the fixed/equilibrium points.

   (c) What is $\lim_{t\to\infty} y(t)$ if $y(0) = 4$?

3. For the system
$$\dot{x} = \sin y,$$
$$\dot{y} = x - x^3,$$
find the fixed/equilibrium points and classify their stability structure.

4. Consider the eigenvalue problem with Robin boundary conditions at both ends:
$$v'' + \lambda v = 0,$$
$$v'(0) - a_0 v(0) = 0, \quad v'(l) + a_l v(l) = 0.$$

   (a) Show that $\lambda = 0$ is an eigenvalue if and only if $a_0 + a_l = -a_0 a_l l$.

   (b) Find the eigenfunctions corresponding to the zero eigenvalue.

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5. For the ODE

$$x^2y'' + xy' - y = 0,$$

   (a) Verify that $y_1(x) = x$ is a solution;

   (b) Find a second, linearly independent solution y_2 using variation of parameters, *i.e.* seek a second solution of the form $y_2(x) = C(x)y_1(x)$;

   (c) Use variation of parameters again with your results from above to solve the non-homogeneous problem

$$x^2y'' + xy' - y = \frac{1}{1-x}.$$

   (Hint: Seek a solution in the form $y = v_1(x)y_1(x) + v_2(x)y_2(x)$.)

6. Find the general solution of

$$(1+x^2)y'' - 4xy' + 6y = 0$$

by considering an expansion about $x = 0$.