1. Solve each of the following initial value problems.

(a) \[ y' = xy^3 (1 + x^2)^{-1/2}, \quad y(0) = 1. \]
(b) \[ y' = e^{2x} + y - 1, \quad y(0) = 0. \]
(c) \[ y'' + 4y = 3 \sin 2x, \quad y(0) = 2, \quad y'(0) = 1. \]

2. Consider the ODE

\[ \dot{y} = e^{-y} \sin y. \]

(a) What are the fixed/equilibrium points?
(b) Determine the stability of the fixed/equilibrium points.
(c) What is \( \lim_{t \to \infty} y(t) \) if \( y(0) = 4? \)

3. For the system

\[ \begin{align*}
\dot{x} &= \sin y, \\
\dot{y} &= x - x^3,
\end{align*} \]

find the fixed/equilibrium points and classify their stability structure.

4. Consider the eigenvalue problem with Robin boundary conditions at both ends:

\[ \begin{align*}
v'' + \lambda v &= 0, \\
v'(0) - a_0 v(0) &= 0, \\
v'(l) + a_l v(l) &= 0.
\end{align*} \]

(a) Show that \( \lambda = 0 \) is an eigenvalue if and only if \( a_0 + a_l = -a_0 a_l. \)
(b) Find the eigenfunctions corresponding to the zero eigenvalue.
5. For the ODE
\[ x^2y'' + xy' - y = 0, \]
(a) Verify that \( y_1(x) = x \) is a solution;
(b) Find a second, linearly independent solution \( y_2 \) using variation of parameters, \( i.e. \) seek a second solution of the form \( y_2(x) = C(x)y_1(x) \);
(c) Use variation of parameters again with your results from above to solve the non-homogeneous problem
\[ x^2y'' + xy' - y = \frac{1}{1-x}. \]
(Hint: Seek a solution in the form \( y = v_1(x)y_1(x) + v_2(x)y_2(x) \).)

6. Find the general solution of
\[ (1+x^2)y'' - 4xy' + 6y = 0 \]
by considering an expansion about \( x = 0 \).