1. (9 pts) Limits:
   (a) Give an example of a limit of a function that you think is best evaluated by plugging
       the limit point into this function. Evaluate this limit.
   (b) Give an example of a limit of a function that you think is best evaluated using L’Hospital’s
       rule (where algebraic simplifications are not sufficient). Evaluate this limit.
   (c) Give an example of a limit of a function that you think is best evaluated using L’Hospital’s
       rule for a different indeterminate form as in (b). Evaluate this limit.

2. (8 pts) Give an example of a use of the fundamental theorem of Calculus to calculate
   (a) a derivative
   (b) an integral.

3. (5 pts) Give a word problem such that an equation containing the derivative of a quantity
   may be obtained from the word description. Clearly indicate the notation you use.

4. (9 pts: 5,4) A line in the xy-plane is given by the equation \( mx + b - y = 0 \).
   (a) Derive a formula for the Euclidean distance between a point \((x_0, y_0)\) and the line
       (denote it \(d(m, b, x_0, y_0)\)).
   (b) Give an equation describing the curve made of all the points that are located at an equal
       distance between the line \( mx + b - y = 0 \) and the point \((x_0, y_0)\).

   For my curiosity (no points), do you know what is the curve obtained above?

5. (6 pts) Prove the formula \( \frac{d(x^n)}{dx} = nx^{n-1} \), for \( n \in \mathbb{N} \).

6. (9 pts) Give an example of an integral where the best technique to use is:
   (a) Substitution
   (b) Partial fractions
   (c) Integration by parts

7. (5 pts) Give an example of a Series which is convergent but not absolutely convergent.

8. (12 pts) Determine as accurately as you can the value of the following finite sums or infinite
   Series.
   \[
   \begin{align*}
   (a) \sum_{n=4}^{30} \left( -\frac{4}{3} \right)^n & \quad (b) \sum_{n=1}^{\infty} \left( \frac{n!}{n^n} \right) & \quad (c) \sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^{2n}}{(2n)!}
   \end{align*}
   \]

9. (6 pts) Explain the connection between Taylor Series and second derivative tests when classifying
   local extrema.
10. (12 pts) Consider the “heart” shape $H$ with boundaries given by $y = 2x - 4$, $y = -2x - 4$, $y = \sqrt{1 - (x - 1)^2}$, and $y = \sqrt{1 - (x + 1)^2}$.
   
   (a) Sketch $H$.
   
   (b) Set up an integral to determine the mass of $H$ if its density is given by $\rho = 1 + x^2 + 2y^2$ (do not integrate).
   
   (c) Determine the area of $H$.

11. (9 pts) The integral $M = \iiint_V h(x,y,z) \, dV$ calculates the mass of a grapefruit.
   
   (a) Explain in words the meaning of $V$, $h(x,y,z)$, and $dV$.
   
   (b) Assuming that the fruit is a sphere centered at the origin, with radius 6, set up the integral for its mass in cylindrical coordinates.
   
   (c) In the same conditions, set up the integral for the mass in spherical coordinates.

12. (11pts) Consider the surface given by $4x + 2y + 3z - 24 = 0$.
   
   (a) Give a point on this surface.
   
   (b) Give two vectors parallel to this surface (but not parallel to each other).
   
   (c) Give a parametrization of this surface.
   
   (d) Give an expression for the flux through this whole (infinite) surface of the vector field $\vec{F} = \langle \frac{1}{x+y+z+1}, \frac{1}{x+y+z+1}, \frac{1}{x+y+z+1} \rangle$. DO NOT EVALUATE THE INTEGRAL.

13. (6 pts) Give a line integral (and its domain) that is easier to compute using Green’s theorem than using a direct parametrization.

14. (21 pts) Answer the following in no more than two lines of text or computations.
   
   (a) Give an equation satisfied by all the points $(x, y, z)$ such that any vector connecting $(x, y, z)$ to the point $(3, 5, 1)$ is perpendicular to the vector $-\vec{i} + 5\vec{j} - 2\vec{k}$.
   
   (b) Give a situation where the dot product is useful.
   
   (c) Give the best local constant approximation, and, separately, linear approximation to $f(x, y) = x^2 \sin y + 2y + 3$, at the point $(1, 0)$.
   
   (d) Consider a function $T(t, z)$ representing the temperature outside this classroom as a function of time, $t$, and elevation, $z$. Explain in words the meaning of $\frac{\partial T}{\partial z}$ and give its expected sign.
   
   (e) If $\vec{F} = \langle y, x^2 \rangle$, what is the maximum of $||\vec{F}||$ over the curve $\vec{r}(t) = \langle t, t^2 \rangle$, for $1 \leq t \leq 2$?
   
   (f) Sketch a 2D vector field whose divergence at the origin is positive.
   
   (g) Parametrize a cylinder of radius 2 centered on the line $\vec{r}(t) = \langle t, 2t, 3t \rangle$. 