Duration: 4 Hours
Instructions: Show your work, credit will not be given to answers without explanation. Partial credit will be awarded for correct work, unless otherwise specified. When you are asked to explain yourself, please write clearly and use complete sentences. Good luck!

When you are asked to *explain* your reasoning, you must use complete sentences.

1. Find and provide a sketch showing all values of the following:
   (a) i^i
   (b) $(-1)^{\sqrt{2}}$
   (c) $(1+i)^{2-i}$

2. Prove the following:
   (a) If $z \neq 1$, show that :
   $$1 + z + z^2 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z}.$$
   (b) Use part (a) to prove Lagrange's Identity
   $$1 + \cos(\theta) + \cos(2\theta) + \cdots + \cos(n\theta) = \frac{1}{2} + \frac{\sin\left(\left(n + \frac{1}{2}\right)\theta\right)}{2\sin(\theta/2)}$$
   where $0 \leq \theta \leq 2\pi$.

3. Consider the following function
   $$f(z) = \frac{(z\,\mathrm{Re}(z))}{|z|} \text{ when } z \neq 0 \text{ and } f(0) = 1.$$
   Is it continuous at 0? Either prove it is or explain why it isn't.

4. Let C be the right half-circle of radius 2 beginning at the point $-2i$ and ending at $2i$. Find:
   $$\int_C z^{1/2} dz$$
   where $z^{1/2}$ is the principal branch.

5. Let f be a nonconstant analytic function in the closed disk $R = \{z : |z| \leq 1\}$. Suppose that $|f(z)| = K$ for all z on the circle of radius 1. Prove that f must have a 0 in the interior of R.

6. Find the Maclaurin series for $f(z) = e^z \cos(z)$ and state where it is guaranteed to converge.

7. Compute the following integrals:
   (a)
   $$\int_{-\infty}^{\infty} \frac{x}{(x^3 - 8)} dx$$

(b)
$$\int_0^{2\pi} \frac{\cos(2\theta)}{5-4\cos(\theta)}\,d\theta.$$

8. Determine the function $T(x,y)$ describing the steady state temperatures in the first quadrant ($x>0$, $y>0$) that satisfies the following boundary conditions:

$$\begin{aligned} T(x,0) &= 10 \qquad \text{for } x>1, \\ T(x,0) &= 20 \qquad \text{for } 0<x<1, \\ T(0,y) &= 20 \qquad \text{for } 0\le y<1, \\ T(0,y) &= 10 \qquad \text{for } y>1. \end{aligned}$$