Instructions: Credit will awarded mainly based on the level of work and explanation.

1. (18 points, 6 points each) Answer each of the following questions.

   (a) Solve the IVP
   $$y'(x) = y^n, \quad y(n) = n - 2 \, ,$$
   where n is a natural number greater than 1.

   (b) Find $y^2(x)$, where $y(x)$ is the solution of the IVP
   $$yy' + f(x) = 0, \quad y(0) = y_0 \, .$$

   (c) Let $f(y) : \mathbb{R} \to \mathbb{R}$ be a differentiable and monotonically increasing function. Find the general solution of
   $$f'(y)y'(x) = xf(y)$$
   and explain why the assumption of monotonicity is necessary.

2. (6 points) Use the substitution $u = \frac{1}{y}$ to find the general solution of
   $$ty' = y - y^2 \, .$$

3. (8 points). Consider the integral problem
   $$y(x) = \int_0^x [1 + y^2(s)] \, ds \, .$$

   (a) (6 points) Find $y(x)$ by converting the problem to an ODE IVP.

   (b) (3 point) What can you say about the uniqueness of $y(x)$?

4. (12 points, 6 points each) Answer each of the following questions.

   (a) Find the general solution of $\ddot{x} + x = e^{it}$.

   (b) Give an example of two 2x2 linear homogeneous systems of ODEs that have the same eigenvalues but different general solutions.

5. (18 points) The rabbits (R) and bobcats (B) of Merced meet at city hall and decide to regulate their populations according to

$$\begin{aligned} \dot{R} &= (B-20)R^2 \,, \\ \dot{B} &= -(R-20)B \,. \end{aligned}$$

   (a) (8 points) Find the equilibrium solutions and sketch the phase portrait.

   (b) (4 points) Explain what happens if initially $R(0) = 60$ and $B(0) = 30$.

   (c) (6 points) Linearize the system around an equilibrium solution. What can you conclude about the stability of that solution?

6. (12 points, 6 points each) Consider the ODE

$$(1-x^2)y'' - xy' + cy = 0 \,, \quad x \in (-1,1) \,, \quad c \in \mathbb{R} \,.$$

   (a) Classify the singular points as regular or irregular.

   (b) For which values of c is there a 2^{nd}-degree polynomial solution?

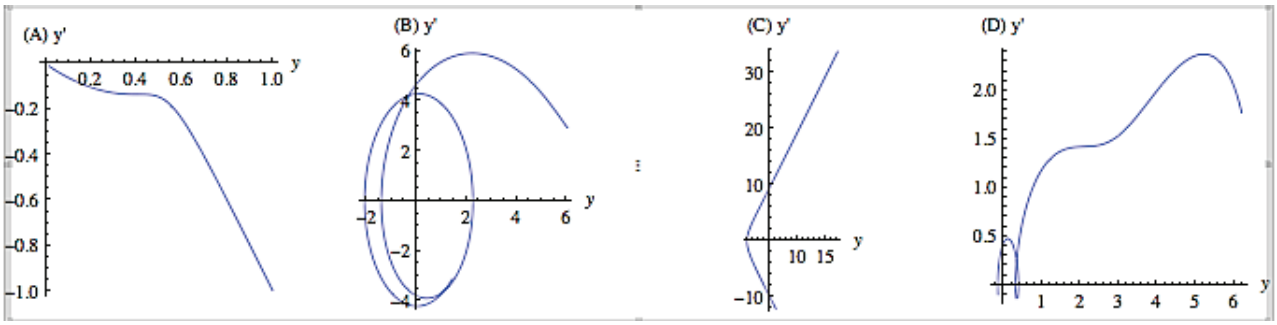
7. (12 points) Match each of the ODEs with a phase-plane solution. Only your final answer will be graded.

   (1) $y'' + 2y' + y = e^{-t}$

   (2) $y'' - 2y' + y = \cos(2t)$

   (3) $y'' + 4y = e^{t}$

   (4) $y'' - 4y = \cos(2t)$



GOOD LUCK!