



Pulsing and Photosynthesis: Numerical Simulations of Soft Corals

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Introduction

- Soft corals in the family Xenidiidae actively pulse their tentacles
- Pulsing was thought to help with food capture
- In the field no food found in their gastric cavity
- Experimental results indicate that pulsing facilitates photosynthesis of the symbiotic algae [1]

Research Goals

- Use numerical methods to model coral pulsing, fluid flow, and photosynthesis of symbiotic algae
- Vary parameters to get insight into mixing and photosynthesis dynamics



Credit [1]

Modeling

Incompressible fluid flow

Solving for fluid velocity $\underline{u}(\underline{x}, t)$ and pressure $p(\underline{x}, t)$

$$\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} + \nabla p = \frac{1}{Re} \nabla^2 \underline{u} + \underline{f}$$

$$\nabla \cdot \underline{u} = 0$$

Immersed boundary method [2]

Solving for coral location $\underline{X}(s, t)$

$$\frac{\partial \underline{X}}{\partial t}(s, t) = \int_{\Omega} \underline{u}(\underline{x}, t) \delta(\underline{x} - \underline{X}(s, t)) d\underline{x}$$

$$\underline{f}(\underline{x}, t) = \int_0^{\ell} \underline{F}(s, t) \delta(\underline{x} - \underline{X}(s, t)) ds$$

Photosynthesis Modeling

Solving for oxygen concentration $c(\underline{x}, t)$ [3]

$$\frac{\partial c}{\partial t} + \underline{u} \cdot \nabla c = \frac{1}{Pe} \nabla^2 c + \int_0^{\ell} \kappa(s, t) \delta(\underline{x} - \underline{X}(s, t)) ds$$

Oxygen-limited photosynthesis model

$$\kappa(s, t) = a(1 - C(s, t)) \quad C(s, t) = \int_{\Omega} c(\underline{x}, t) \delta(\underline{x} - \underline{X}(s, t)) d\underline{x}$$

Relevant Parameters

Biological Parameters

parameter	description	value	units
L	tentacle length	0.4070	cm
γ	pulsation frequency	0.5286	s^{-1}
ν	kinematic fluid viscosity	0.01	$cm^2 s^{-1}$
D	diffusion coefficient	2×10^{-5}	$cm^2 s^{-1}$

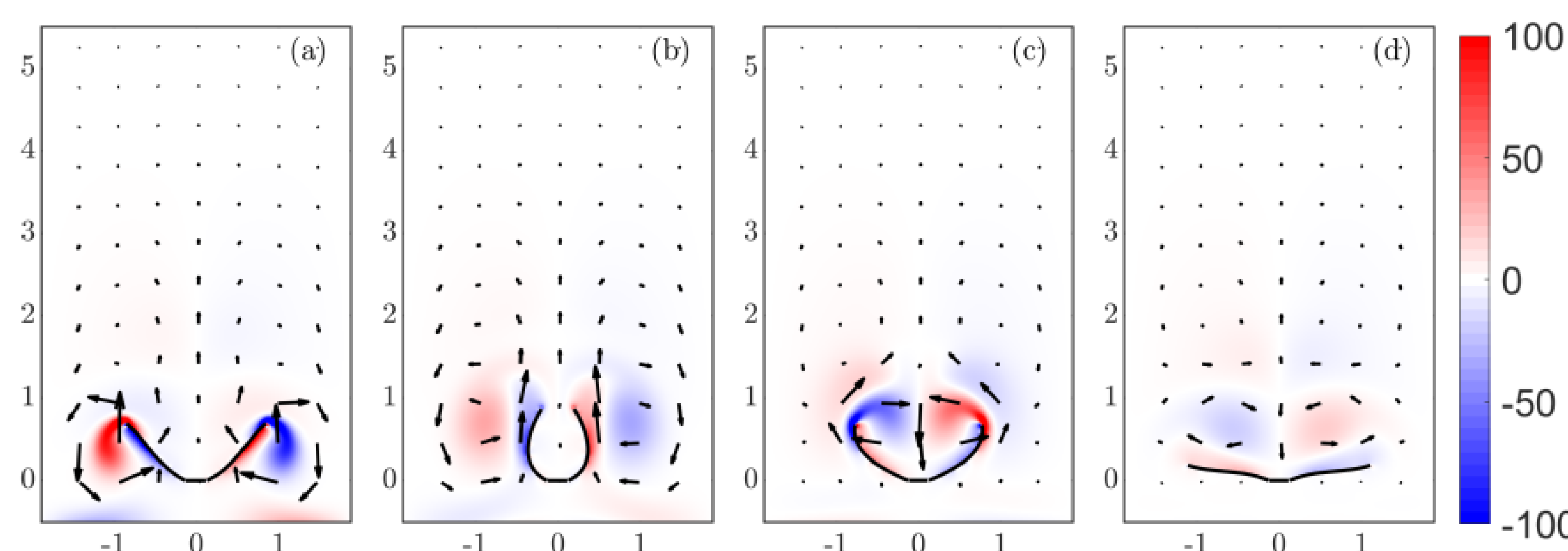
Dimensionless Parameters

$$\text{Reynolds number} \quad Re = L^2 \gamma / \nu$$

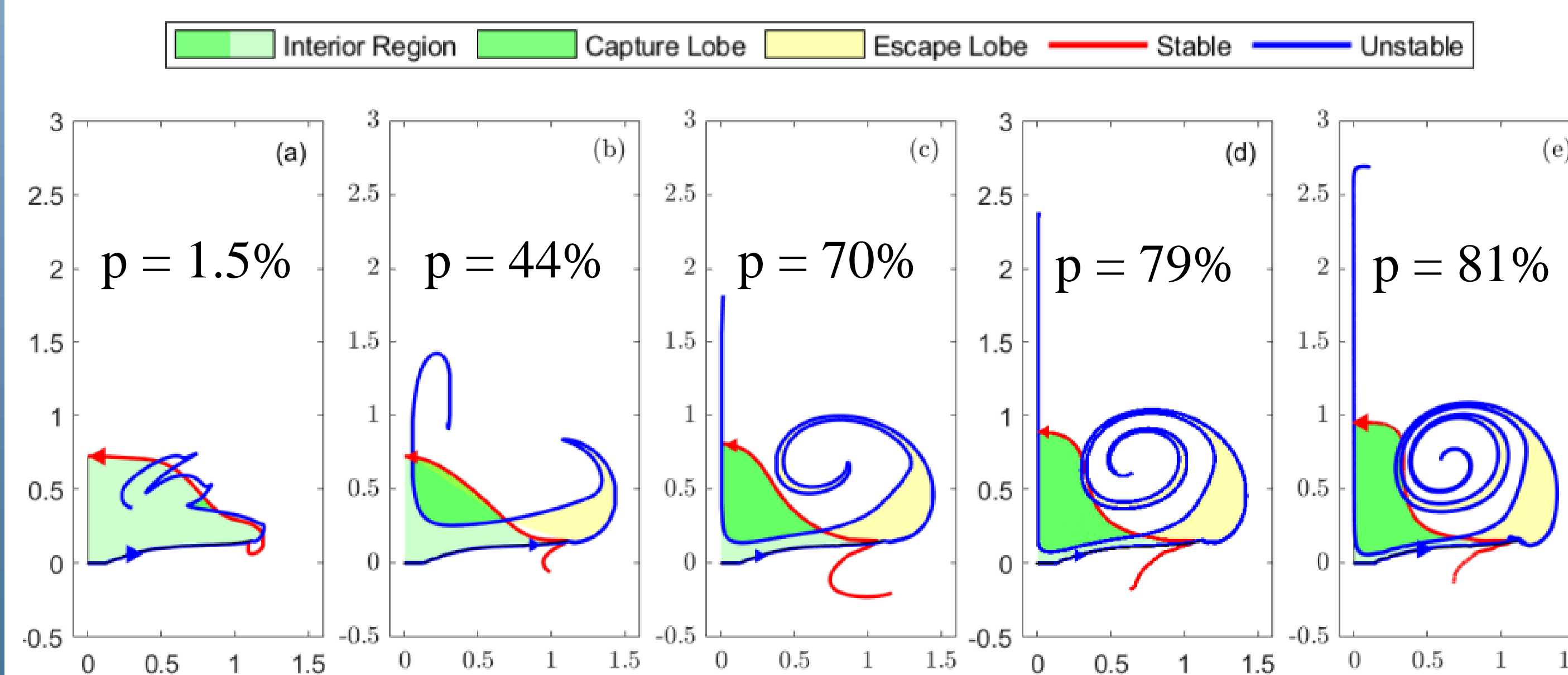
$$\text{Péclet number} \quad Pe = L^2 \gamma / D$$

Varying the Reynolds and Péclet number to investigate the role of fluid inertia and diffusivity in mixing and photosynthesis

Mixing Analysis



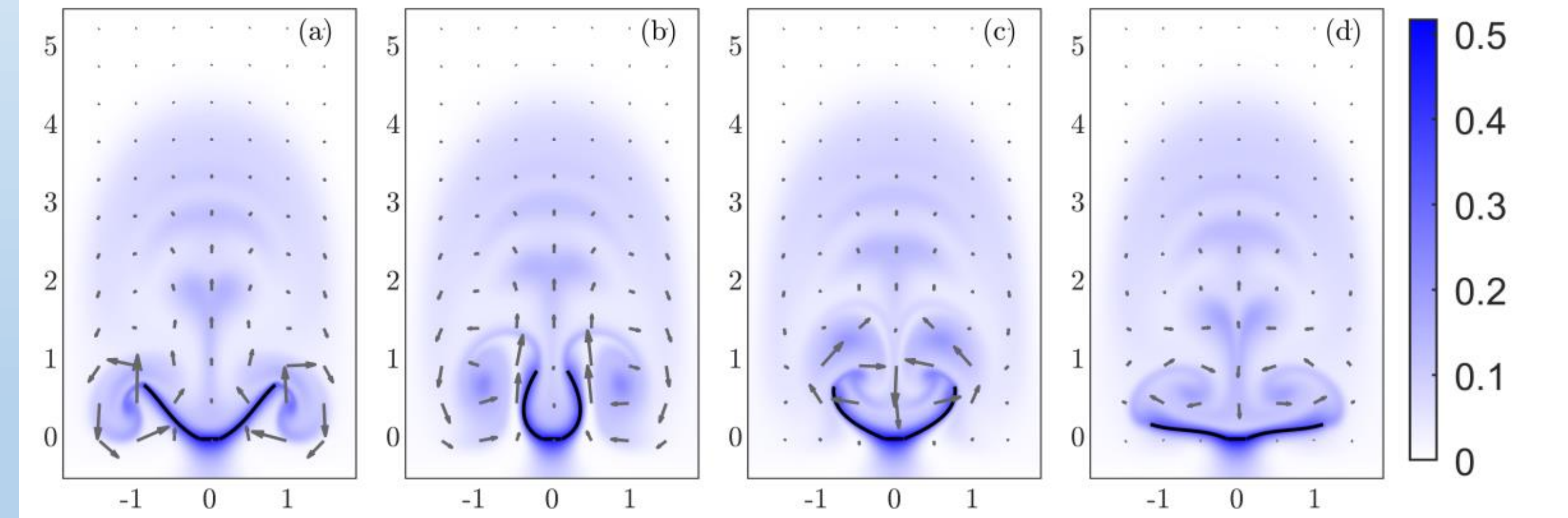
The fluid flow of a pulsing soft coral at $Re=8$ at (a) 10%, (b) 30%, (c) 50%, and (d) 80% of a pulse. The color map shows the vorticity, and the vectors give the velocity field in the simulation.



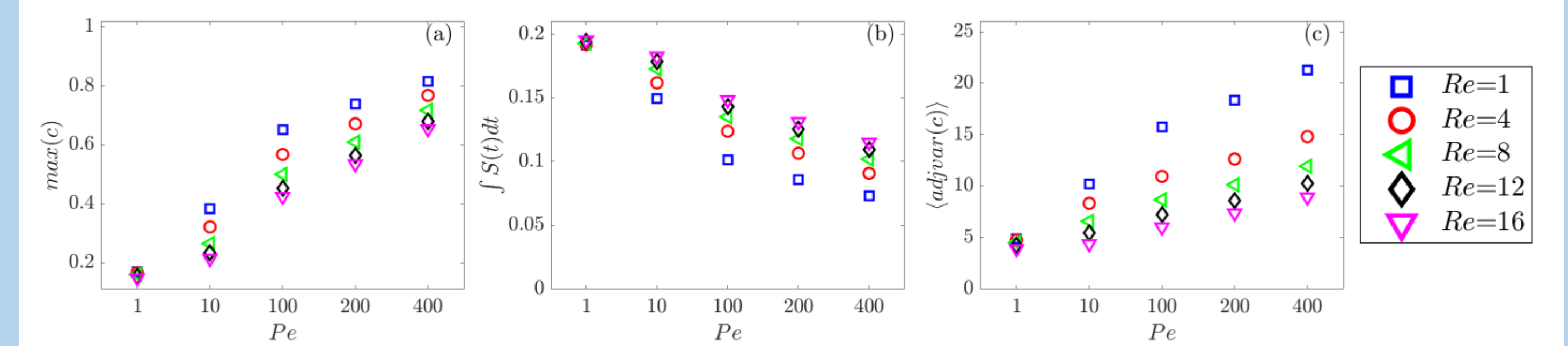
Analysis of Poincaré maps for (a) $Re=1$, (b) $Re=4$, (c) $Re=8$, (d) $Re=12$, and (e) $Re=16$. The stable manifold (red) and unstable manifold (blue) are plotted as well as the location of the tentacle (black). The interior regions, capture lobes, and escapes lobe are denoted with different colors.

$$\% \text{ of fluid entering the interior region} \quad p = \frac{\text{area of capture lobe}}{\text{area of interior region}} \times 100$$

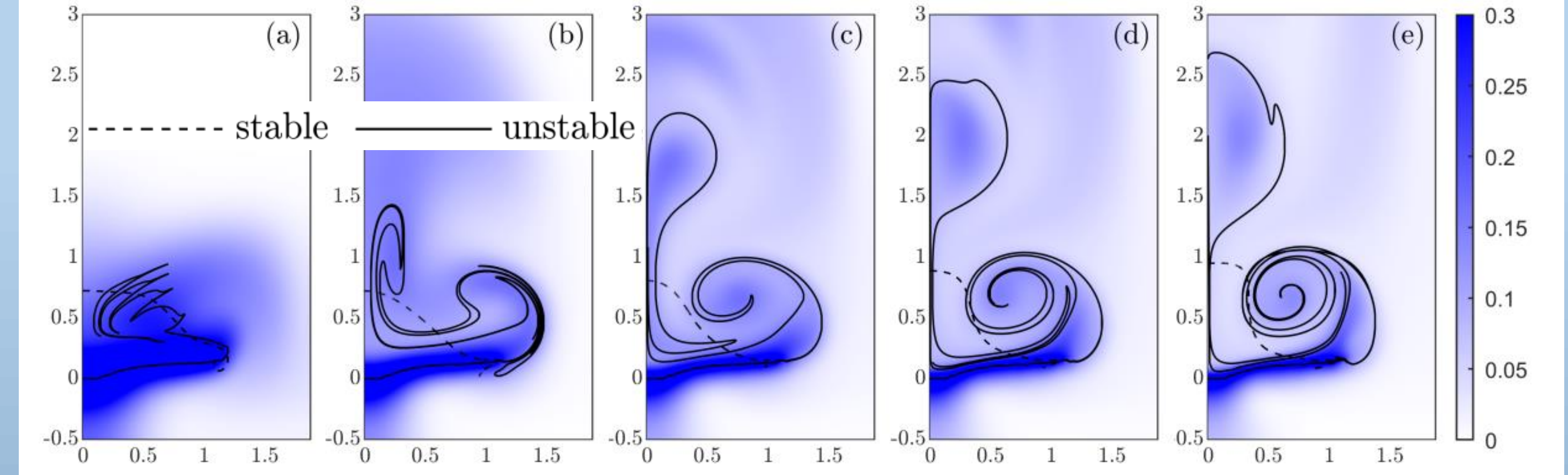
Concentration Dynamics



The concentration dynamics of the oxygen-limited model with $Re=8$ and $Pe=100$ at (a) 10%, (b) 30%, (c) 50%, and (d) 80% through the tenth pulse.



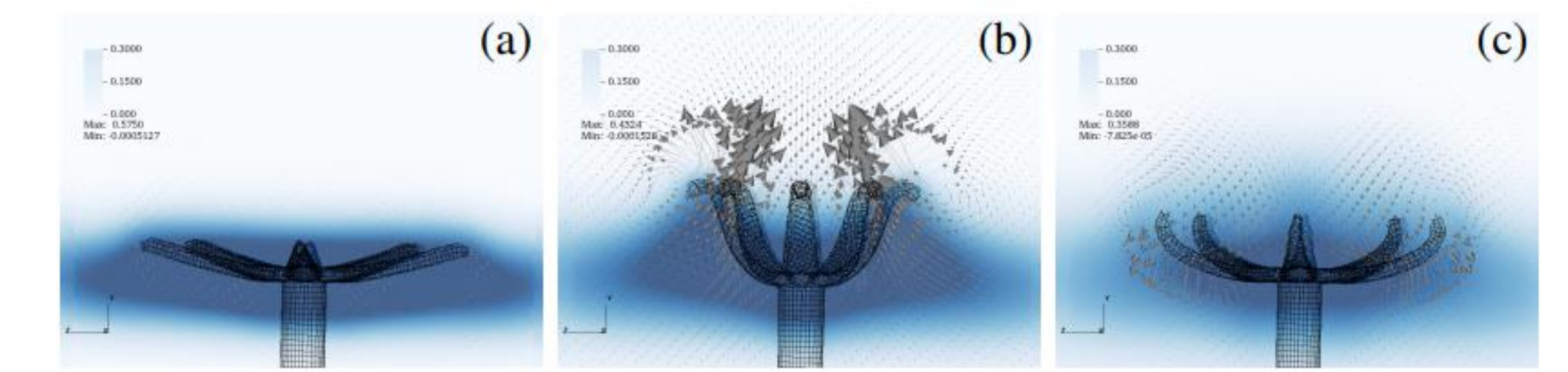
(a) Maximum concentration, (b) total oxygen produced, and (c) temporal average of the adjusted concentration variance during the tenth pulse for varying Péclet and Reynolds numbers for the oxygen-limited model.



Oxygen concentration for the oxygen-limited photosynthesis model at the end of ten pulses for (a) $Re=1$, (b) $Re=4$, (c) $Re=8$, (d) $Re=12$, and (e) $Re=16$ overlaid with the corresponding stable (dashed) and unstable (solid) manifolds.

Future Work

- 3 dimensional simulations using IBFE [4]



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