

- 1] (a) F (b) T (c) F (d) T (e) F

$$\begin{aligned}
 2] \frac{d}{dx} \sin x &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h} \\
 &= \lim_{h \rightarrow 0} \left[\sin(x) \frac{\cos(h) - 1}{h} + \cos(x) \frac{\sin(h)}{h} \right] \\
 &= \sin(x) \lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} + \cos(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h} \\
 &= \cos(x)
 \end{aligned}$$

$$3] (a) h'(x) = \pi^3 x^{x^2-1} + \ln(\pi^3) (\pi^3)^x + \pi$$

$$(b) \frac{d}{d\theta} \ln[\theta \cos \theta] = \frac{1}{\theta \cos \theta} \frac{d}{d\theta} [\theta \cos \theta] = \frac{\cos \theta - \theta \sin \theta}{\theta \cos \theta}$$

$$\begin{aligned}
 (c) \frac{d}{dx} e^{[5^x + x \ln x]} &= e^{[5^x + x \ln x]} \frac{d}{dx} [5^x + x \ln x] \\
 &= e^{[5^x + x \ln x]} (\ln(5)5^x + \ln x + 1)
 \end{aligned}$$

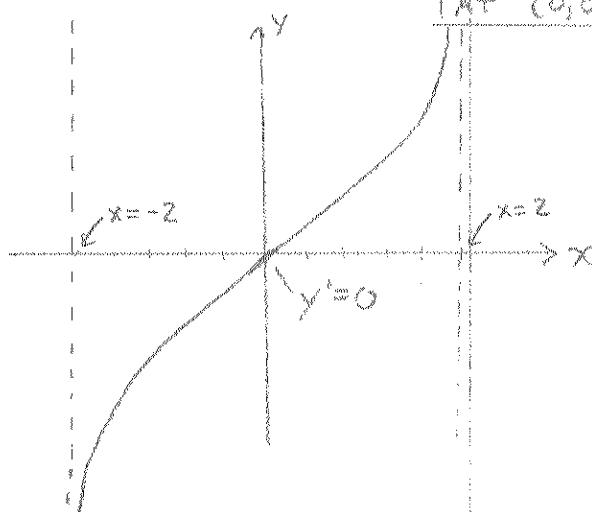
$$(d) y = \frac{x}{y+a} \Rightarrow y' = \frac{d}{dx} [x(y+a)^{-1}] = [(y+a)^{-1} - x(y+a)^{-2} y]$$

$$y' \left[1 + x(y+a)^{-2} \right] = \frac{1}{y+a} \quad y' = \frac{1}{(y+a) \left[1 + \frac{x}{(y+a)^2} \right]}$$

$$y' = \frac{-(y+a)}{(y+a)^2 + x} \quad y' \Big|_{x=0, y=0} = \frac{1}{a}$$

SLOPE OF
PERP LINE = $-a$
AT $(0,0)$

- 4] TAKEN FROM EXAM #1



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$$(a) \frac{dg}{dx} = \begin{cases} 2x-1 & -\infty < x \leq 1 \\ \ln(x) & 1 < x < 1.5 \end{cases}$$

$\frac{dg}{dx}$ is UNDEFINED AT $x=1$, AND ZERO AT $x = \frac{1}{2}$

CRITICAL POINTS $x=1, x=\frac{1}{2}$

INTERVALS

$$(-\infty, \frac{1}{2}) : g'(x) = -1 < 0 \quad \text{DECREASING} \quad \left. \begin{array}{l} \\ \end{array} \right\} x = \frac{1}{2}, \text{ LOCAL MIN}$$

$$(\frac{1}{2}, 1) : g'(\frac{3}{4}) = \frac{3}{2} - 1 > 0 \quad \text{INCREASING} \quad \left. \begin{array}{l} \\ \end{array} \right\} x = \frac{3}{4}$$

$$(1, 1.5) : g'(x) > 0 \quad \text{INCREASING}$$

(b) g'' UNDEFINED AT $x=1$; NONZERO FOR $x \neq 1$

$$(-\infty, 1) \quad g'' = 2 > 0 \quad \text{CONCAVE UP} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{NO INFLECTION POINTS}$$

$$(1, 1.5) \quad g'' = \frac{1}{x} > 0 \quad \text{CONCAVE UP}$$

(c) SKETCH
SOME VALUES

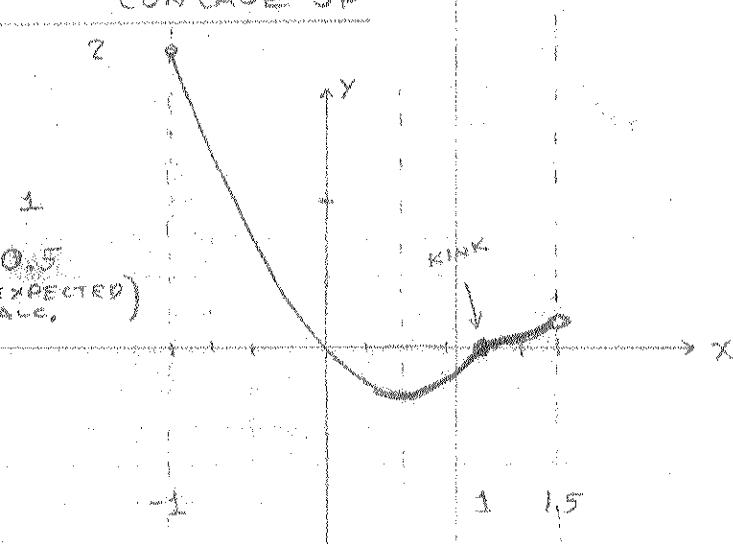
$$g(0) = 0 \quad g(1) = 1.5 \ln(1.5) \approx 0.5$$

$$g(1) = 0 \quad \approx 0.1 \quad (\text{NOT EXPECTED TO CALC.})$$

$$g(-1) = 2$$

$$g(\frac{1}{2}) = -\frac{1}{4}$$

$$\left. \frac{dg}{dx} \right|_{x=-1} = -3 \quad \left. \frac{dg}{dx} \right|_{x=0} = -1$$



(d) $g(\frac{1}{2})$ is A GLOBAL MIN; NO GLOBAL MAXIMA

$$6(a) \tilde{g}(x) = g(-1) + g'(-1)[x - (-1)]$$

$$\tilde{g}(x) = 2 + (-3)(x+1) = -2 - 3x - 3$$

$$g(x) = -3x - 1$$

$$(b) E(x) = g(x) - \tilde{g}(x)$$

$$E(1.2) = 1.2 \ln(1.2) - 0.2 + 3(1.2) + 1$$

$$= 1.2 \ln(1.2) + 4.4$$

5.6