1 (a) F (b) F (c) F (d) T (e) T

2 (FROM EXAM 2)

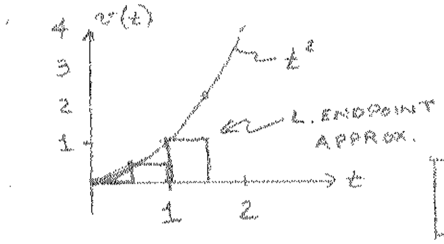
$$\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h} = \lim_{h\to 0}\frac{\sin(x)\cos(h)+\sin(h)\cos(x)-\sin(x)}{h}$$

$$= \lim_{h\to 0}\left[\sin(x)\frac{\cos(h)-1}{h} + \cos(x)\frac{\sin(h)}{h}\right]$$

$$= \sin(x)\lim_{h\to 0}\frac{\cos(h)-1}{h} + \cos(x)\lim_{h\to 0}\frac{\sin(h)}{h} = \underline{\underline{\cos(x)}}$$

3

(a) DISTANCE TRAVELED $= \int_0^{1.5} t^2\,dt \approx (0)^2\Delta t + (1/2)^2\Delta t + (1)^2\Delta t$



$$= 0 + \frac{1}{4}\cdot\frac{1}{2} + 1\,\frac{1}{2} = \frac{1}{8} + \frac{1}{2} = \boxed{\frac{5}{8}}$$

STARTING POSITION

$$\boxed{\text{POSITION AT } t = 3/2 \approx 1 + 5/8 = \frac{13}{8}}$$

(b) $\int \frac{(x+1)}{x^2}\,dx = \int (x^{-1} + x^{-2})\,dx = \boxed{\ln(x) - x^{-1} + C}$

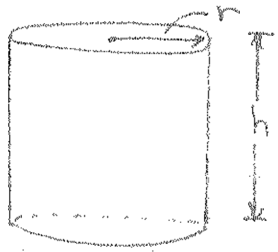
(c) $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin(x)}\right) = \lim_{x\to 0}\frac{\sin(x)-x}{x\sin(x)} \overset{\text{L'HOPITAL}}{=} \lim_{x\to 0}\frac{\cos(x)-1}{\sin(x)+x\cos(x)}$

$$= \lim_{x\to 0}\frac{-\sin(x)}{\cos(x)+\cos(x)-x\sin(x)} = \boxed{0}$$

(d) $\int_0^4 g(x)\,dx = \int_0^1 g(x)\,dx + \int_1^2 g(x)\,dx + \int_2^4 g(x)\,dx$

$$= -\tfrac{1}{2}(1)(1) + \tfrac{1}{2}(1)(1) + 2\,(1) \Rightarrow \boxed{\int_0^4 g(x)\,dx = 2}$$

4



CYLINDER VOLUME: $V = \pi r^2 h = 1\ m^3$

SIDEWALL COST PER UNIT AREA: α

SIDEWALL AREA: $A_S = 2\pi r h$

LID AREA: $A_L = 2\pi r^2$

TOTAL COST: $C = \alpha A_S + 2\alpha A_L$

$$= \alpha 2\pi r h + \alpha 4\pi r^2$$

FROM VOLUME REQUIREMENT $\Rightarrow$ $h = \frac{1}{\pi r^2}(m^3)$

$$C = \alpha 2\pi r\left(\frac{(m^3)}{\pi r^2}\right) + \alpha 4\pi r^2 = \alpha 2 r^{-1}(m^3) + \alpha 4\pi r^2$$

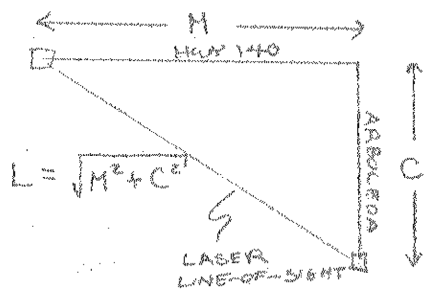
$$\frac{dC}{dr} = -\alpha 2 r^{-2}(m^3) + \alpha 8\pi r \qquad \frac{d^2C}{dr^2} = \alpha 4 r^{-3}(m^3) + \alpha 8\pi$$

(BECAUSE $\frac{d^2C}{dr^2} > 0$ FOR $r > 0$, $C(r)$ IS CONCAVE UP $\Rightarrow$ LOCAL MIN)

CRITICAL POINTS

$$\frac{dC}{dr} = 0 \Rightarrow -2(m^3) + 8\pi r^3 = 0 \qquad \boxed{r = \left(\frac{1}{4\pi}\right)^{1/3} m}$$

5



WANT $\frac{dH}{dt}$ WHEN $H = 3$ miles AND $C = 1$ mile

GIVEN $\frac{dC}{dt} = -50$ mph, $\frac{dL}{dt} = -70$ mph

$$\frac{dL}{dt} = \frac{1}{2}\left(H^2 + C^2\right)^{-1/2}\left(2H\frac{dH}{dt} + 2C\frac{dC}{dt}\right)$$

PLUG IN KNOWN VALUES

$$-70\text{mph} = \frac{1}{2}(9+1)^{-\frac{1}{2}}\frac{1}{\text{miles}}\left(2(3\text{miles})\frac{dH}{dt} + 2(1\text{mile})(-50\text{mph})\right)$$

$$= \frac{1}{2\sqrt{10}}\left(6\frac{dH}{dt} - 100\text{mph}\right)$$

$$\boxed{\frac{(-140\sqrt{10} + 100)\text{mph}}{6} = \frac{dH}{dt}}$$

$\left(\frac{dH}{dt} \simeq -57\text{ mph}\right)$

(APPROACHING INTERSECTION AT ABOUT 57 mph)