**UC Merced: MATH 21 — Final Exam — 19 December 2005**

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your instructor's name (Sprague) and (4) a grading table. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. There are a total of eight problems and a total of 150 points. Please start each of the eight problems on a new page.

1. **(20 points)** Answer the following Always True (T) or False (F). Only your final answer will be graded on these problems.

   (a) All continuous functions are integrable.

   (b) You **cannot** use the fundamental theorem of calculus to evaluate $\displaystyle\int_{-3}^{-2} \frac{1}{x}\,\mathrm{d}x$ because $\ln(x)$ is defined only for $x > 0$.

   (c) $\dfrac{\mathrm{d}}{\mathrm{d}\theta}\left[\cosh^2\left(2\theta^3\right) - \sinh^2\left(2\theta^3\right)\right] = 0$

   (d) When profit is maximized, marginal cost is equal to marginal revenue.

   (e) Using Newton's law of gravity, we determine that the attraction force between two particular objects at a distance $R$ is $F$. Their attraction force at a distance of $2R$ is $F/2$.

2. **(15 points)** Use the definition of the derivative along with the Mean-Value Theorem for Definite Integrals to show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_a^x f(u)\mathrm{d}u = f(x)$$

   where $a$ is a constant and $f$ is continuous.

   *The MVT for Definite Integrals*: If $g$ is continuous on $[a, b]$, then at some point $c$, where $a \le c \le b$, $\quad g(c) = \dfrac{1}{b-a}\displaystyle\int_a^b g(x)\mathrm{d}x$.

3. (20 points total) Find the requested information in the following limit problems:

   (a) **(6 points)** $\displaystyle\lim_{\theta\to 0} \theta \sin\left(\frac{1}{\theta}\right)$

   (b) **(8 points)** $\displaystyle\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin(x)}\right)$

   (c) **(6 points)** Show that $\displaystyle\lim_{x\to 0}\frac{\sin(x)}{x} = 1$

4. **(10 points)** Suppose that you're on the surface of some planet, where you can breath and the acceleration constant is $g = -2\ \mathrm{m/s^2}$. You throw a 1 kg grapefruit directly into the air with an initial velocity of 2 m/s. Assume that it leaves your hand at 2 m from the ground.
   (a) How long until it hits the ground?
   (b) When does it reach its maximum height?
   (c) Name one physical effect that is neglected in the mathematical model used to solve this problem.

5. (25 points total) Find the requested information in the following differentiation problems:

   (a) **(8 points)** Find $h'(t)$ for $h(t) = t^{\pi^3} + (\pi^3)^t + \pi t$

   (b) **(8 points)** Evaluate $\frac{\mathrm{d}}{\mathrm{d}\theta}\ln[\theta\sin(\theta)]$

   (c) **(9 points)** Find $m'(z)$ if $m(z) = \displaystyle\int_1^{e^z}\frac{\cos(t^2)}{\sqrt{t+2}}\,\mathrm{d}t$

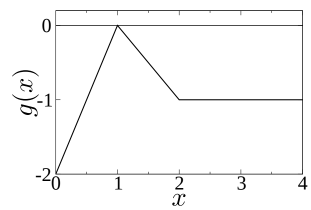
6. (30 points total) Find the requested information in the following integration problems:

(a) **(8 points)** Evaluate $\displaystyle\int_{-\pi/2}^{\pi/2} |\sin(\theta)|\ \mathrm{d}\theta$

(b) **(6 points)** Evaluate $\displaystyle\int \frac{(z+1)}{z^2}\ \mathrm{d}z$

(c) **(8 points)** Evaluate $\displaystyle\int \frac{\sin(x)}{\sqrt{\cos(x)+5}}\ \mathrm{d}x$

(d) **(8 points)** Given the plot of $g(x)$ given below, calculate $\displaystyle\int_0^4 g(x)\ \mathrm{d}x$

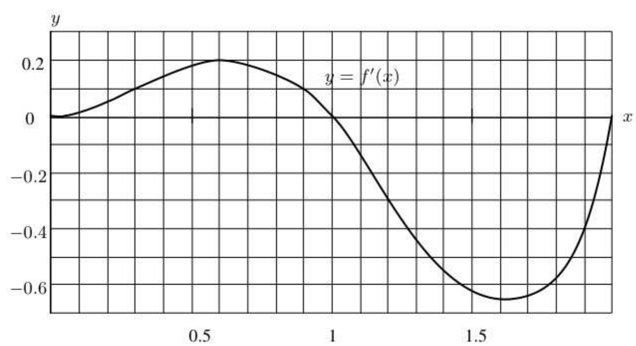



7. **(10 points)** Consider the graph shown below for $y = f'(x)$, and answer the following three questions. Only your final answers will be graded.

(a) On what intervals is $f$ **increasing**?

(b) On what intervals is the graph of $f$ **concave up**?

(c) Which value is greater: $f(0.25)$ or $f(1)$?




8. **(20 points)** I wish to make a window that lets the maximum amount of light through, but reduces the harsh light when the sun is high in the sky. To this end, I want my window to look like the figure below: it will be composed of a rectangle of clear glass with a semicircle end made of tinted glass. A unit area of tinted glass admits half of the light emitted by a unit area of clear glass. What is the **width of the rectangular portion** that will allow the most light to pass through the window. The perimeter of the window is fixed at 10 m.