

- On the front of your blue book print (1) your name, (2) your student ID number, (3) your discussion section number, and (4) a grading table.
- Show all work in your blue book and **BOX IN YOUR FINAL ANSWERS** where appropriate.
- Please start each problem on a new page. There are a total of seven problems on both sides of this paper and a total of 100 points.
- NO books, notes, crib sheets, calculators or any other electronic devices are allowed.

Only the final answer will be graded for problems 1 and 2. No justification is needed.

- (20 points, 4 points each) Determine whether the following statements are True or False.
 - If an object moves with the same average velocity over every time interval, then its average velocity equals its instantaneous velocity at any time.
 - If $f'(x) = g'(x)$ for all real number x , then $f(x) = g(x)$.
 - The sinusoidal function $y = -3\sin(4x) + 5$ completes 4 cycles in the interval $[0, 2\pi]$.
 - For sufficiently large values of x , $f(x) = 1000x^3 + 345x^2 + 17x + 394$ is less than $g(x) = 0.01x^4$.
 - If $\lim_{x \rightarrow 3} f(x) = 7$ and $g(3) = 4$, then $\lim_{x \rightarrow 3} (f(x) + g(x)) = 11$.
- (30 points, 6 points each) Choose A, B, C, D, or E for each of the following questions.
 - Which of the following functions have an inverse?

(I) $\cos x$ with domain $[0, 1]$	(II) $e^{-(x-1)}$	(III) $(x-2)^2$ with domain $(-\infty, 1]$		
A) II only	B) I and II only	C) I and III only	D) III only	E) I, II and III
 - Which of the following functions are increasing functions?

(I) the derivative of an increasing function	(II) the derivative of a concave up function			
(III) the inverse of an increasing function	(IV) the inverse of a concave up function			
A) I and III only	B) II and III only	C) II and III only	D) I and IV only	E) III and IV only
 - The graph of a function $g(x)$ is given below. Which of the following statements about its derivative $g'(x)$ are true?

- | | |
|----------------------------------------------|-------------------------------------------------------|
| (I) $g'(0) = 0$. | (II) $g'(x)$ is an odd function. |
| (III) $g'(x)$ is decreasing over $(-1, 1)$. | (IV) $g'(x)$ has vertical asymptotes at $x = \pm 1$. |

- | | | | | |
|-----------|------------------|-------------------|--------------------|----------------------|
| A) I only | B) I and II only | C) I and III only | D) II and III only | E) I, II, III and IV |
|-----------|------------------|-------------------|--------------------|----------------------|

(d) Which of the following statements are true?

- (I) If $f(x)$ is not continuous at $x = a$, then it is not differentiable at $x = a$.
 (II) If $f(x)$ is not differentiable at $x = a$, then it is not continuous at $x = a$.
 (III) If $f(x)$ is differentiable at $x = a$, then it is continuous at $x = a$.

- A) II only B) I and II only C) I and III only D) III only E) I, II and III

(e) Consider the logarithmic function $f(x) = c \ln(kx)$, where $c < 0$ and $k > 0$ are constants. The graph of $f(x)$ is

- A) increasing and concave up. B) decreasing and concave up. C) increasing and concave down.
 D) decreasing and concave down.

Show your reasoning clearly for problems 3–7. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit.

3. (10 points) Consider the piecewise function $f(x)$ defined below. Can you find a value for b such that $f(x)$ is continuous at $x = 2$. If yes, find this value. If not, explain why.

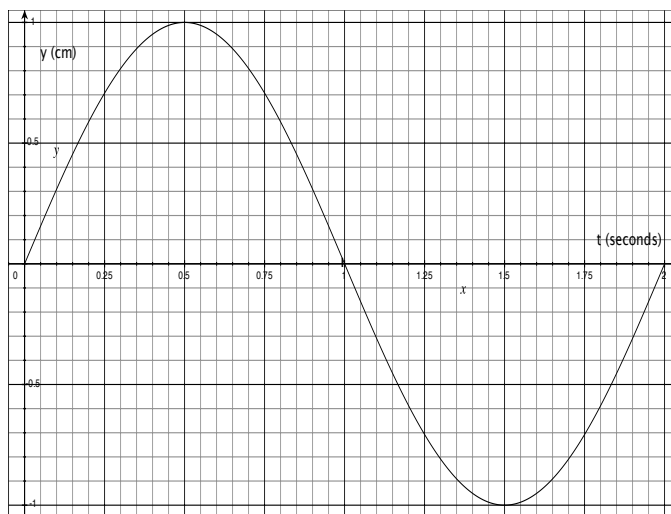
$$f(x) = \begin{cases} \cos\left((x-1)\frac{\pi}{2}\right) \frac{x-2}{|x-2|}, & \text{for } x \neq 2 \\ b, & \text{for } x = 2. \end{cases}$$

4. (8 points) Use the Intermediate Value Theorem to show that the equation $e^x = x + 2$ has a solution on the interval $[0, 2]$.

5. (10 points) $g(x) = \frac{1}{1-x}$. Using the definition of a derivative, find $g'(x)$.

6. (10 points) What is the y -intercept of the tangent line to $m(x) = \frac{5x^3 + 1}{x}$ at $x = -1$?

7. A block attached to the end of a spring is moving vertically along the y -axis around $y = 0$. The graph below shows its y -coordinate as a function of time t .



(a) (3 points) When (over what time interval(s)) is this block above $y = 0$?

(b) (3 points) When (over what time interval(s)) is this block moving upward?

(c) (6 points) Is $\left. \frac{d^2y}{dt^2} \right|_{t=0.5}$ positive or negative? What are its units? What is its practical meaning?