Show your reasoning clearly for problems 1–3. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit.

1. Solve the following unrelated problems

   (a) (10 points) Write down, but **DO NOT** evaluate, a Riemann sum to estimate the definite integral \( \int_{0}^{1} \sqrt{\tan(x)} \, dx \) using \( n = 3 \) subintervals and right-endpoints.

   (b) (10 points) Calculate the exact area enclosed between the \( x \)-axis and the parabola \( y = 1 - x^2 \).

   (c) (10 points) Find the limit \( \lim_{x \to 0} \left( \frac{1}{\ln(1 + x)} - \frac{1}{x} \right) \).

2. (15 points) A field will be made in the shape of a rectangle with an area of 1500 square feet. It is surrounded on all four sides by walls and is divided into three smaller rectangles by fences (see picture). The wall costs $15 per foot and the fence costs $10 per foot. What is the lowest possible cost to build such a field?

   ![Figure 1: Problem 2](image1.png)

   ![Figure 2: Problem 3](image2.png)

3. (15 points) A police cruiser, approaching a right–angled intersection from the North, is chasing a speeding car that has turned the corner and is now moving straight East. When the cruiser is 0.6 miles North of the intersection and the car is 0.8 miles to the East, the officer determines with his radar that the distance between himself and the speeding car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?
4. (20 points, 4 each) Determine whether the following statements are true or false. Write out the whole word "TRUE" or "FALSE" for each problem.

(a)  \[ \int e^x \sqrt{1 + e^x} \, dx = \frac{2}{3} \left(1 + e^x \right)^{3/2} + C. \]

(b)  \[ \frac{d}{dx} \left[ \int_{1}^{5} \frac{x^2 + 1}{x} \, dx \right] = 0. \]

(c)  \[ \frac{d}{dx} \left[ \int_{x}^{\infty} \sin(t) \, dt \right] = 2x \sin(x^2) - \sin(x). \]

(d)  If \( f(x) \) is an even function and \( \int_{0}^{1} f(x) \, dx = 2 \), then \( \int_{-1}^{1} f(x) \, dx = 4. \)

(e) The definite integral of a function \( f(x) \) has the same units as the function \( f(x) \) itself.

5. (20 points, 4 each) The graph of the derivative of \( F(x) \) is given below. Determine whether the following statements about \( F(x) \) are true or false. Write out the whole word "TRUE" or "FALSE" for each problem.

(a) \( F(x) \) is constant between \( x = -2 \) and \( x = -1. \)

(b) \( F(x) \) has a local maximum at \( x = 5. \)

(c) \( F(x) \) is concave down between \( x = -1 \) and \( x = 1. \)

(d) \( F(x) \) has inflection points at \( x = 3 \) and \( x = 4. \)

(e) \( F(2) = F(-2). \)