

UC Merced: MATH 21 - Exam #1 - 16 February 2006

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your instructor's name (Bianchi), (4) your section number, and (5) a grading table (see front board). Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators, and crib sheets are not permitted. There are a total of five problems on both sides of this paper and a total of 100 points. Please start each of the five problems on a new page.

1. (20 Points) Answer the following Always True (T) or False (F). Only your final answer will be graded on these problems.

- (a) If a function is NOT continuous at $x = a$, it CANNOT be differentiable at $x = a$.
- (b) If $\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)$, then $f(x)$ is continuous at $x = a$.
- (c) $\frac{d}{dx} [3^x] = x3^{x-1}$.
- (d) If $f''(x) < 0$ for all $x > 0$, then $f'(5) > f'(7)$.
- (e) As x increases to 100, $f(x) = 1/x$ gets closer to 0, so the limit as x goes to 100 of $f(x)$ is 0.

2. (14 Points) Find the requested limits, if they exist. If they do not exist, explain.

- (a) $\lim_{x \rightarrow -1} \sqrt{x+1}$
- (b) $\lim_{x \rightarrow \infty} \frac{3x^2 - 2}{x^3 - 2x + 1}$

3. (41 Points Total) Answer the following unrelated questions. Please use sufficient blue-book space so that your solutions are clear.

- (a) (7 Points) Find $\frac{df}{dx}$ if $f(x) = \begin{cases} 3x^2 + 5x - 3, & x \leq 2 \\ \frac{1}{x}, & x > 2 \end{cases}$
- (b) (10 Points) Find the equation of the tangent line to $f(x) = 4\sqrt[3]{x} - (x - 5)^2$ at the point $x = 8$.
- (c) (10 Points) Using the definition of derivative, find $g'(u)$ where $g(u) = \frac{5}{u-3}$. Where is $g(u)$ NOT differentiable?
- (d) (7 Points) Suppose that $P(q)$ gives the profit in dollars of selling q T-shirts. Explain in terms a child would understand (no math terms) what $P'(100) = 5$ means. What are the units of $P'(100)$?
- (e) (7 Points) Let $f(x) = 4x^3 - 2x^2 + 5x - 7$. Where is the graph of $f(x)$ concave up?

4. (15 Points) Draw a possible graph of $y = f(x)$ if $f(x)$ has all of the following properties (your answer should be a single graph).

- $f(x)$ is defined on the interval $[-2, 4]$.
- $f'(x) > 0$ for $x < 0$.
- $f'(x) = 0$ for $0 < x < 1$.
- $f'(x) < 0$ for $1 < x$.
- $f'(x)$ is undefined at $x = 0$ and $x = 1$.
- $f''(x) > 0$ for $x < 0$.
- $f''(x) < 0$ for $x > 1$.

There's more on the next page!

5. (10 Points) Graph $y = f'(x)$ using the graph of $y = f(x)$ below.

