Math 21, Fall 2007 Exam 2 solution

1 (a) False (b) True (c) False (d) False (e) False

2 (a) $L(x) = f(1) + f'(1)(x-1) = 3 + 1(x-1) = \boxed{x+2}$

$f(0.98) \approx 0.98 + 2 = \boxed{2.98}$

(b) $f'(x)$ exists for all x in the domain

$f'(x) = 0$ at $x = 0, 2 \Rightarrow$ critical pts are $\boxed{x=0, x=2}$

$f'(x)$ does not change sign at $x = 0 \Rightarrow$ $\boxed{x=0 \text{ is neither local max or local min}}$

$f'(x)$ changes from + to − at $x = 2 \Rightarrow$ $\boxed{x=2 \text{ local max}}$

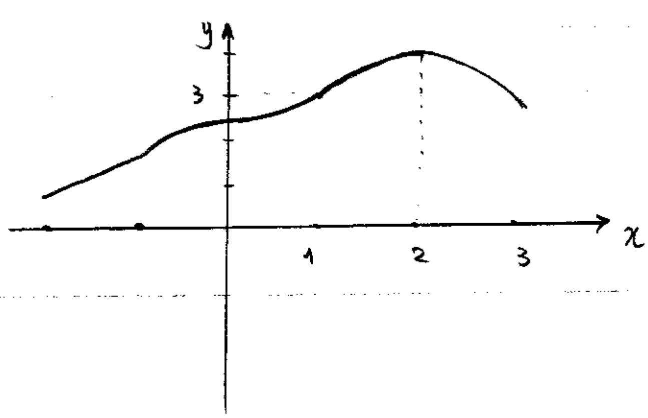
(c) $f''(x) > 0$ on $(0,1)$ because $f'(x)$ is increasing there

$\Rightarrow \boxed{f(x) \text{ is concave up on } (0,1)}$

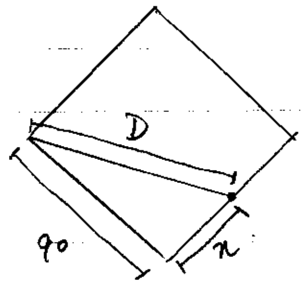
$f''(x) < 0$ on $(-1,0)$ and $(1,3)$ because $f'(x)$ is decreasing

$\Rightarrow \boxed{f(x) \text{ is concave down on } (-1,0) \text{ and } (1,3)}$

(d)



3.



$$D^2 = 90^2 + x^2$$

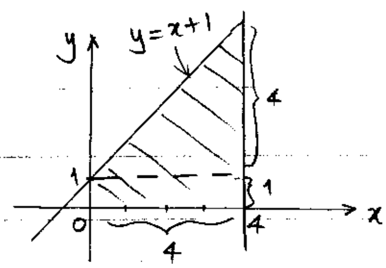
$$2D\frac{dD}{dt} = 0 + 2x\frac{dx}{dt}$$

$$D\frac{dD}{dt} = x\frac{dx}{dt}$$

$x = 45$ ft, $D = 45\sqrt{5}$ ft, $\frac{dx}{dt} = 24$ ft/s $\Rightarrow$

$$\cancel{45}\sqrt{5}\,\frac{dD}{dt} = \cancel{45}(24) \Rightarrow \boxed{\frac{dD}{dt} = \frac{24}{\sqrt{5}} \text{ ft/s}}$$

4. (a)

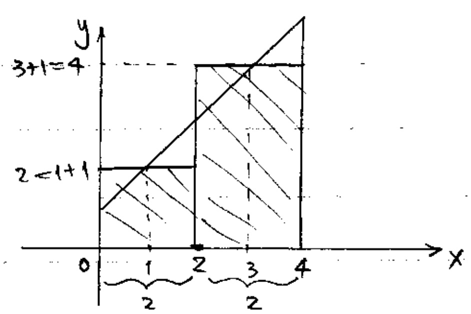


$$\int_0^4 (x+1)\,dx = \text{area of shaded region}$$

$$= \frac{1}{2}\cdot 4\cdot 4 + 4\cdot 1$$

$$= 8 + 4 = \boxed{12}$$

(b)



$$\int_0^4 (x+1)\,dx \approx$$

$$2\cdot(2) + 2(4) = 4 + 8 = \boxed{12}$$