Fall 2008 Math 21 midterm 1 Solution

1 (a) $\frac{d}{dx}\sqrt{x^2+1} = \frac{1}{2}(x^2+1)^{-1/2}(2x) = x(x^2+1)^{-1/2} = \frac{x}{\sqrt{x^2+1}}$

(b) $\frac{d}{dx}\left(\sin(x^2)\cos x\right) = \cos(x^2)(2x)\cos x + \sin(x^2)[-\overset{\sin}{\cancel{\cos}}\,x]$

$= 2x\cos(x^2)\cos x - \sin(x^2)\overset{\sin}{\cancel{\cos}}(x)$

2 (a) $g'(x) = 3x^2 - 2$, slope $= g'(2) = 3(2^2) - 2 = 10$

(b) $y - 5 = 10(x-2)$

(c) $y = 0 \Rightarrow 0 - 5 = 10(x-2) \Rightarrow -\frac{1}{2} = x - 2 \Rightarrow x = 2 - \frac{1}{2} = \frac{3}{2}$. The line intersects x-axis at $(\frac{3}{2}, 0)$

3 (a) $\lim_{x\to 5^-} \frac{x^2-4x-5}{x^2-3x-10} = \lim_{x\to 5^-} \frac{(x-5)(x+1)}{(x-5)(x+2)} = \lim_{x\to 5^-} \frac{x+1}{x+2} = \frac{5+1}{5+2} = \frac{6}{7}$

(b) $\lim_{x\to 4^+} \frac{x+4}{x-4}$: As x gets arbitrarily close to 4 from the right, $x+4$ gets arbitrarily close to 8 and $x-4$ arbitrarily close to, but larger than, 0. Then $\frac{x+4}{x-4}$ gets arbitrarily large. so

$\lim_{x\to 4^+} \frac{x+4}{x-4} = +\infty$

4 (a) $f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{h^2\sin(\frac{1}{h}) - 0}{h} = \lim_{h\to 0} h\sin(\frac{1}{h})$

For any h, $-1 \le \sin(\frac{1}{h}) \le 1 \Rightarrow -|h| \le h\sin(\frac{1}{h}) \le |h|$.

$\lim_{h\to 0} -|h| = 0$ and $\lim_{h\to 0} |h| = 0 \Rightarrow \lim_{h\to 0} h\sin(\frac{1}{h}) = 0$ by the Squeeze theorem. So $f'(0) = 0$

(b) $f(x) = x^2\sin(\frac{1}{x})$ is continuous at $x \neq 0$. At $x = 0$, since $f'(0)$ exists, $f(x)$ is differentiable and so continuous. $\Rightarrow$ $f(x)$ is continuous on $(-\infty, +\infty)$.

5.