① (a) $\frac{d}{dx}\sin(x+y^2) = \cos(x+y^2)\cdot\left(1+2y\cdot\frac{dy}{dx}\right)$

$\Rightarrow \cos(x+y^2)\cdot\left(1+2y\cdot\frac{dy}{dx}\right) = \frac{dy}{dx}$

$\Rightarrow \cos(x+y^2) = \left(1-2y\cos(x+y^2)\right)\cdot\frac{dy}{dx}$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{\cos(x+y^2)}{1-2y\cos(x+y^2)}}$

(b)

For $(x,y)=(\pi,0)$,

$$\frac{dy}{dx} = \frac{\cos\pi}{1-2\cdot 0\cdot\cos\pi} = -1.$$

[For fake answer: $\frac{dy}{dx} = \frac{\pi+\cos 0}{2+0^2} = \frac{\pi+1}{2}$]

Tangent line is $y = -(x-\pi)$
$= \pi - x$

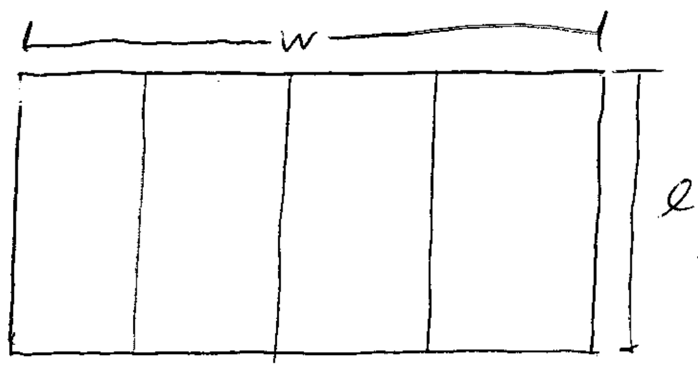
[$y = \frac{\pi+1}{2}(x-\pi)$]

Approximation

$$y \approx -(3-\pi) = .1416\ldots$$

$$\left[\ y \approx \frac{\pi+1}{2}(3-\pi)\ \right]$$

(2)



Maximize $A = lw$

l, w have relation $5l + 2w = 710$

$\Rightarrow w = 355 - \frac{5}{2}l$

$\Rightarrow A = l \cdot (355 - \frac{5}{2}l)$

$\Rightarrow \frac{dA}{dl} = 355 - 5l$

Maximum A occurs when $355 - 5l = 0$

or $l = 71$

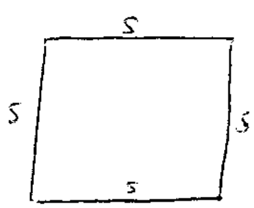
$\Rightarrow w = 355 - 2.5\ 71 = 177.5$

$\Rightarrow \boxed{A_{max} = 177.5 \cdot 71 = 12{,}602.5\ ft^2}$

(3)

$$\lim_{x \to \infty} \frac{2 - 3x^2}{5x^2 + 4x} = \lim_{x \to \infty} \frac{\frac{2}{x^2} - 3}{5 + \frac{4}{x}} = \boxed{-\frac{3}{5}}$$

(4)



$P = 4s, \quad A = s^2$

$dP/dt = 4\, ds/dt$

$dA/dt = 2s \cdot ds/dt$

When $P = 40cm$, $dP/dt = 5$; $s = 10cm$, $ds/dt = 5/4 cm$

$\Rightarrow dA/dt = 2 \cdot 10cm \cdot \frac{5}{4} cm/min = \boxed{25 cm^2/min}$

5

$a(t) = \cos t \Rightarrow v(t) = \sin t + C$

since $v(0) = 1$, $\sin 0 + C = 1 \Rightarrow C = 1$

$\Rightarrow v(t) = \sin t + 1$

$\Rightarrow s(t) = -\cos t + t + C'$

since $s(0) = 0$, $-\cos 0 + 0 + C' = 0 \Rightarrow C' = 1$

$\Rightarrow \boxed{s(t) = -\cos t + t + 1}$

6

(a) df/dx changes sign from negative to positive at $x=3$, so at $x=3$ there is a local minimum

(b) f is concave up/down when df/dx is increasing/decreasing. So f is concave up on $[-1, 0]$, $[2, +\infty)$; it is concave down on $[0, 2]$

(c)