

UC Merced: MATH 21 — Exam #3 — 09 May 2008

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your discussion section number and instructor's name (Sprague or Lei), (4) a grading table, and (5) your **seat number**. Show all work in your bluebook and **BOX IN YOUR FINAL ANSWERS** where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. There are a total of xxx problems on one side of this paper and a total of 50 points. Please start each problem on a new page. You have 50 minutes to complete the exam.

Some potentially useful information:

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}; \quad \frac{d}{dx} \arccos x = \frac{-1}{\sqrt{1-x^2}}; \quad \frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

For certain conditions, the following is true: $(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$

1. (6 points: 2 each) Answer the following Always True or False. Only your final answers will be graded on these problems.

(a) If $f(x)$ is increasing and concave up, then $f^{-1}(x)$ is increasing and concave down.

(b) $\lim_{x \rightarrow 1^+} \frac{x}{\ln x} = \lim_{x \rightarrow 1^+} \frac{1}{\frac{1}{x}} = \frac{1}{\frac{1}{1}} = 1$

(c) $\sin^{-1}(\sin 3\pi/4) = 3\pi/4$

2. (24 points total) Evaluate the following expressions and simplify your answers when possible.

(a) (5 points) $\tan(\cos^{-1}(3/5))$

(b) (5 points) $e^{-2 \ln 3}$

(c) (7 points) $\int \frac{e^x}{1+e^{2x}} dx$

(d) (7 points) $\lim_{x \rightarrow 0^+} x \ln x$

3. (7 points) In our class, $\ln x$ is defined to be $\ln x = \int_1^x \frac{1}{t} dt$. Use a Riemann sum with 2 rectangles and the Midpoint rule to estimate $\ln 5$.

4. (7 points) Use the derivative formula $(2^x)' = (\ln 2)2^x$ and the fact that $\log_2 x$ is the inverse function of 2^x to find $\frac{d}{dx}(\log_2 x)$. (2 points if you write down the answer from memory without justification.)

5. (6 points: 2 each) Evaluate the following expressions. (Hint: Very little calculation is involved. Think conceptually.)

(a) $\frac{d}{dx} \int_x^1 \sinh(t) \cosh(t) dt$

(b) $\frac{d}{dx} \int_0^1 \sinh(x) \cosh(x) dx$

(c) $\int_0^1 \frac{d}{dx} (\sinh(x) \cosh(x)) dx$