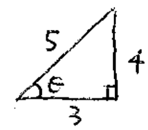
Math 21, spring 2008 Solutions to Midterm 3

1. (a) TRUE (HW 7.1: 2(c) WS12: 10, 12)

(b) FALSE (HW 7.8: $\boxed{17}$)

(c) FALSE (WS14: 5(e))

2. (a) (HW 7.6: 12, WS14: 5(a, f)) $\theta = \cos^{-1}(3/5) \Rightarrow$



$\Rightarrow \tan(\cos^{-1}(3/5)) = \tan(\theta) = \boxed{\frac{4}{3}}$

(b) (HW 3(a)) $e^{-2\ln 3} = e^{\ln(3^{-2})} = 3^{-2} = \boxed{\frac{1}{9}}$

(c) (WS13: 9) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx \; \frac{u = e^x}{\frac{du}{dx} = e^x} \; \int \frac{\frac{du}{dx}}{1+u^2}\,dx = \int \frac{du}{1+u^2}$

$= \arctan(u) + C = \boxed{\arctan(e^x) + C}$

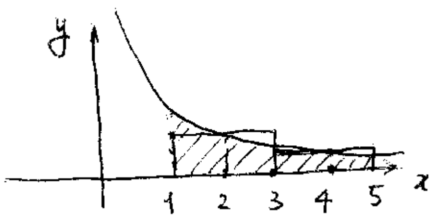
(d) (HW 7.8: 40, Example 6 of 7.8)

$\lim_{x\to 0^+} x\ln x = 0(\ln 0^+) = 0(-\infty) \leftarrow$ indeterminate

$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{1}{x}\cdot\frac{x^2}{1} = \lim_{x\to 0^+} x = \boxed{0}$

L'Hopital's Rule

3. (WS13: 5, HW5.1: 4, 5.2: 2, 5, 9, 10)



$\ln 5 =$ shaded area $\approx \frac{1}{2}\cdot 2 + \frac{1}{4}\cdot 2 = 1 + \frac{1}{2} = \boxed{\frac{3}{2}}$

4. (HW7.4: 34, WS14: 7, proof of formula 3 in 7.6, example 4 of 7.7)

2^x and $\log_2 x$ are inverse to each other $\Rightarrow 2^{\log_2 x} = x \Rightarrow$

$\frac{d}{dx}[2^{\log_2 x}] = \frac{d}{dx}[x] \Rightarrow (\ln 2)\, 2^{\log_2 x}\cdot\frac{d}{dx}(\log_2 x) = 1 \Rightarrow (\ln 2)\, x \cdot \frac{d}{dx}(\log_2 x) = 1$

$\Rightarrow \frac{d}{dx}(\log_2 x) = \boxed{\frac{1}{(\ln 2)x}}$

5. (FTC) (all problems from 5.3 and 5.4)

(a) $\frac{d}{dx}\int_x^1 \sinh(t)\cosh(t)\,dt = \frac{d}{dx}\left[-\int_1^x \sinh(t)\cosh(t)\,dt\right] = \boxed{-\sinh(x)\cosh(x)}$

(b) $\frac{d}{dx}\int_0^1 \sinh(x)\cosh(x)\,dx = \frac{d}{dx}\left[\text{some number}\right] = \boxed{0}$

(c) $\int_0^1 \frac{d}{dx}\left(\sinh(x)\cosh(x)\right)dx = \left[\text{antiderivative of } \frac{d}{dx}(\sinh(x)\cosh(x))\right]\Big|_0^1$

$$= \sinh(x)\cosh(x)\Big|_0^1$$

$$= \sinh(1)\cosh(1) - \sinh(0)\cosh(0)$$

$$= \boxed{\sinh(1)\cosh(1)}$$