

Duration: 50 minutes, Material Covered: Stewart 3.5-4.9

Instructions: On the front of your blue/green book print (1) your name, (2) your student ID number, (3) your discussion section number and instructor's name (Kristina Crona, Lei, Yue, or Mike Sprague), and (4) a grading table. Show all work in your blue/green book and **BOX IN YOUR FINAL ANSWERS** where appropriate.

Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 50.

Keep your exam! Next week's quiz will be based on 2, 4, 5, and 6, in addition to the two problems from section 5.2.

1. (10 points total, 2 each) Answer the following **Always** True or False and short answer questions. Only your final answers will be graded on these problems.

(a) (short answer) For what values of k does $y = \frac{x^2 + 3x + 5}{4x + 1 + x^k}$ have a horizontal asymptote?

(b) (short answer) If we know that $f'(p) = 0$, can we conclude that f has either a local maximum or a local minimum at $x = p$? (Yes or No.)

(c) (short answer) Water drains from the bottom of a tank. $V(t)$ is the volume of water remaining in the tank after t minutes. Is $V'(10)$ positive or negative and why?

(d) (True or False) If $f(x)$ is continuous on $(0, 1)$, then it reaches both its absolute maximum and minimum somewhere inside $(0, 1)$.

(e) (True or False) A polynomial of degree 3 has at most 3 real roots.

2. (10 points) Find an equation of the tangent line to the curve $x^2y^2 + y^3 = 2$ at the point $(1, 1)$.

3. (5 points) A particle is moving as described by the following equations. $a(t)$ stands for the acceleration, $v(t)$ the velocity, and $s(t)$ the position function of this particle. Find the position function $s(t)$.

$$a(t) = \cos t + \sin t, \quad s(0) = 1, \quad v(0) = 3.$$

4. (5 points) Sketch the graph of a function that satisfies all of the given conditions. Make sure that you label your axes and sketch carefully so that we may clearly identify all important features such as increasing/decreasing, concavity, asymptote(s), symmetry, and etc.

$$f'(2) = 0, f(2) = -1, f(0) = 0,$$

$$f'(x) < 0 \text{ if } 0 < x < 2, f'(x) > 0 \text{ if } x > 2,$$

$$f''(x) < 0 \text{ if } 0 \leq x < 1 \text{ or if } x > 4, f''(x) > 0 \text{ if } 1 < x < 4,$$

$$\lim_{x \rightarrow \infty} f(x) = 0,$$

$$f(-x) = f(x) \text{ for all } x.$$

5. (10 points) Show that of all the rectangles with a given area, the one with smallest perimeter is a square.
6. (10 points) If an ice cube melts so that its surface area decreases at a rate of $10 \text{ cm}^2/\text{min}$, find the rate at which an edge length decreases when the edge is 5 cm long.