

Duration: 50 minutes, Material Covered: Stewart 5.1-7.6

Instructions: On the front of your blue/green book print (1) your name, (2) your student ID number, (3) your discussion section number and instructor's name (Kristina Crona, Lei, Yue, or Mike Sprague), and (4) a grading table. Show all work in your blue/green book and **BOX IN YOUR FINAL ANSWERS** where appropriate. Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 50.

Some potentially useful information:

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}, \quad \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}.$$

1. (10 points total; 2 points each) Answer the following True (T) or False (F) questions. Only your final answers will be graded on these problems.

- (a) A continuous function is differentiable.
 (b) A continuous function is integrable.
 (c)

$$\ln 2 = \int_1^2 \frac{1}{x} dx.$$

- (d) The Midpoint rule states that if one chooses the sample point x_i^* to be the midpoint of the interval, then the resulting Riemann sum is strictly between L_n and R_n (with notation as in the book).
 (e) Let $f(x)$ be a differentiable one-to-one function and let $f^{-1}(a) = b$. If $f'(b) \neq 0$ then

$$(f^{-1})'(a) = \frac{1}{f'(b)}.$$

2. (12 points total; 2 points each) Only your final answers will be graded on these problems. **No motivation is required.**

- (a) Find $\int_0^3 f(x) dx$ if

$$\int_0^1 f(x) dx = 1, \int_1^2 f(x) dx = 2 \text{ and } \int_2^3 f(x) dx = 3.$$

Answer with a **number**.

- (b) Find $\frac{d}{dx}(4^x)$.
 (c) Find one solution to the differential equation $\frac{dy}{dx} = y$.
 (d) Find g^{-1} if $g = f^{-1}$.
 (e) Let $f(x) = \int_0^x e^{t^2} dt$ for $x \geq 0$. Find $f'(2)$, that is $\left. \frac{df}{dx} \right|_{x=2}$. Answer with a **number**.

(f) Let

$$g(x) = \int_0^x e^t \sin t \, dt$$

for $0 \leq x \leq 2\pi$. In particular, the domain of $g(x)$ is $[0, 2\pi]$. Find a number a such $g(x)$ has a global maximum at $x = a$ (in other words $(a, g(a))$ is a global maximum). Answer with the **number** a .

3. (3 points) Let $v(t)$ denote the vertical speed of a peregrin falcon as a function of the time t in minutes. Here $v(t) < 0$ if the falcon descends. The vertical displacement from $t = 0$ to $t = 15$ equals

$$\int_0^{15} v(t) \, dt.$$

Express the vertical distance traveled by the falcon from $t = 0$ to $t = 15$ as an integral. Answer with **the integral**.

Hint: If the falcon first descends 100 meters and then ascends 100 meters, then the vertical displacement is 0 meters and the vertical distance traveled is 200 meters.

4. (5 points) Evaluate the Riemann sum for

$$f(x) = x^2, \quad 0 \leq x \leq 3$$

with three subintervals, taking the sample points to be right endpoints. This Riemann sum approximates the area under the parabola $f(x) = x^2$ from 0 to 3.

5. (15 points) Determine the following indefinite integrals.

(a) $\int \frac{1}{x} \, dx$

(b) $\int x^2 e^{x^3} \, dx$

(c) $\int \frac{3}{1+x^2} \, dx.$

6. (5 points)

$$f(x) = \begin{cases} x^2 + 1 & x < 0 \\ \cos x & x \geq 0. \end{cases}$$

Evaluate the integral $\int_{-1}^{\frac{\pi}{2}} f(x) \, dx.$