Math 22 Unit Exam 1

Instructions. Read each problem carefully and follow all of its instructions. For each of the problems, please write the best solution you can as clearly as you can in your blue book. For short answer and true/false questions, write clearly your answer and any additional explanations needed.

Problems.

1. (10 points) Explain why we must always add an arbitrary constant when computing an indefinite integral.

2. (10 points) WITHOUT COMPUTING ANY NUMERICAL VALUES, put the following approximations to the integral
$$\int_0^1 (x^2+1)\mathrm{d}x$$
and its exact value in order from smallest to largest: LEFT(2), RIGHT(2), MIDPT(2) and TRAP(2).

3. (10 points) Find
$$\int (2x+1)\mathrm{e}^{x^2}\mathrm{e}^x\mathrm{d}x.$$

4. For each statements below, say whether they are true or false. If the statement is false, revise the statement to make it true.

   (a) (2 points) $P(x)$ and $Q(x)$ are two polynomial functions. The degree of $P(x)$ is p and the degree of $Q(x)$ is q. The rational function $R(x) = P(x)/Q(x)$ is proper if $p \leq q$.

   (b) (2 points) If a function f is continuous on the closed interval $[a, b]$, then f is integrable on $[a, b]$.

   (c) (2 points) If f is integrable on $[a, b]$, then
$$\int_b^a f(x)\mathrm{d}x = \int_a^b f(x)\mathrm{d}x.$$

   (d) (2 points) The integral $\int_0^2 (x-1)^{-2}\mathrm{d}x$ is improper.

   (e) (2 points) The integral $\int_0^\infty x^{-5/2}\mathrm{d}x$ converges.

5. (10 points) Using trigonometric substitution, show that

$$\int \sqrt{9-4t^2}\,\mathrm{d}t = \frac{t}{2}\sqrt{9-4t^2} + \frac{9}{4}\arcsin\left(\frac{2t}{3}\right) + C.$$

You may find the following trigonometric identities useful in this computation.

$$\cos^2\theta = \frac{1}{2}[1+\cos(2\theta)], \qquad \sin^2\theta = \frac{1}{2}[1-\cos(2\theta)]$$
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta), \qquad \cos(2\theta) = 1-2\sin^2(\theta)$$

6. (10 points) Find

$$\int \frac{1}{x(x^2+1)}\mathrm{d}x.$$

7. (10 points) For the function $f(x) = x\sin(2x)$, find the function $F(x)$ with the properties that $F'(x) = f(x)$ and also $F(0) = 0$.