

## Math 22 Final Exam

**Instructions.** Read each problem carefully and follow all of its instructions. The test is worth 125 points total.

- (10 points) Find:  $\int \frac{1 + e^{2t}}{e^t} dt$ .
- (10 points) Find the Taylor polynomial of degree 2 that approximates the function  $f(x) = \sqrt{x^3 + 1}$  near  $x = 2$ .
- (10 points) Find the partial fraction decomposition of  $\frac{2x^2 - x + 4}{x^3 + 4x}$ .
- Explain carefully the answers to the following questions. You must give a complete answer to receive full credit.

(a) (5 points) What does it mean for the improper integral

$$\int_1^{\infty} f(x) dx$$

to converge? (Hint: How does one formally interpret this improper integral?)

(b) (5 points) What does it mean for an infinite sequence to converge?

(c) (5 points) What does it mean for an infinite series to converge? (Hint: What is a partial sum of a series?)

(d) (5 points) What does it mean for a rational function to be proper?

(e) (5 points) Suppose  $a_n > a_{n+1} > 0$  and the series  $\sum a_n$  converges. Does the series  $\sum (-1)^n a_n$  converge? Explain why or why not.

- (10 points) Use the substitution  $w = (x^4 + 3)^2$  to evaluate

$$\int x^3 (x^4 + 3)^5 e^{(x^4 + 3)^2} dx.$$

6. (10 points) Solve the initial value problem

$$\frac{dy}{dx} = x(y^2 + 4), \quad y(0) = 233.$$

7. (10 points) Find the volume of the solid obtained by revolving one arc ( $0 \leq x \leq \frac{1}{3}$ ) of the curve  $y = \sin(3\pi x)$  about the  $x$ -axis. You must show your work to receive full credit.

8. (10 points) Find:  $\int [\sin^6(x) \cos(x) + \cos^6(x) \sin(x)] dx$ .

9. (10 points) Does the integral

$$\int_2^{\infty} \frac{d\theta}{\sqrt{\theta^3 + 1}}$$

converge or diverge? You must give a complete justification of your answer. (Hint: You do not need to evaluate the integral to answer the question)

10. (10 points) Find the exact value of  $\sum_{k=0}^{\infty} \frac{5^k + 1}{11^k}$ . (Hint: Consider breaking up this series into a sum of two series and compute each of their values)

11. (10 points) Find the radius of convergence of the power series

$$f(x) = 1 + x + 4x^2 + 9x^3 + 16x^4 + 25x^5 + 36x^6 + \dots$$