Math 22: Unit 2 Exam

Spring Semester 2006

Instructions. Read each problem carefully and follow all of its instructions. For each of the problems below, write a clear and concise solution in your blue book. For any short answer questions, write clearly your answer and any additional explanation that is needed.

1. (10 points) Find the exact value of the volume of the solid obtained by revolving the region above the graph of $y = x^2$ and below the line $y = 9$ about the line $y = 9$.

2. Answer the following questions.

   (a) (5 points) For what values of p does the integral $\displaystyle\int_1^\infty t^{-p}\,\mathrm{d}t$ converge?

   (b) (5 points) For what values of p does the integral $\displaystyle\int_0^1 t^{-p}\,\mathrm{d}t$ converge?

   (c) (5 points) Does the integral $\displaystyle\int_0^\infty \frac{\mathrm{d}y}{1+e^y}$ converge?

3. A cylindrical form is filled with a slow curing concrete. The base of form is 10 ft in radius, and the height is 25 ft. While the concrete hardens, gravity causes the density to vary from a density of $90\,\mathrm{lb}/\mathrm{ft}^3$ at the bottom to a density of $50\,\mathrm{lb}/\mathrm{ft}^3$ at the top.

   (a) (5 points) By assuming that the density varies linearly from top to bottom, find the function that gives the density at height h from the bottom.

   (b) (5 points) Consider a horizontal slice of the cylindrical form with thickness Δh at height h. Find an approximation to the weight of this slice.

   (c) (5 points) Derive a definite integral for the total weight of the resulting concrete column and then compute the total weight.

4. (10 points) Find the length of the curve given parametrically as $x = \sin 2t$ and $y = \cos 2t$ for $0 \le t \le 2\pi$.